Speaker: Matthew De Courcy-Ireland

Title: Markoff graphs mod p

Abstract:

We discuss a family of cubic surfaces defined by

$$x^2 + y^2 + z^2 = xyz + k$$

modulo prime numbers. The solutions form a graph, where each vertex (x, y, z) is joined to the other solution of the same quadratic in any of the three variables. These moves are related to a nonlinear action of the modular group $\mathrm{PGL}(2,\mathbb{Z})$ on the surface. We outline some ways these equations arise, and how we became interested in showing that the associated graphs cannot be embedded in the plane. We describe constructions showing that the graphs for $k = 0$ are not planar if the prime is congruent to 1 modulo 4, or congruent to a quadratic residue 1, 2, or 4 modulo 7. We also sketch a proof of non-planarity for all sufficiently large primes.