Calculus I Final Exam Review Problems

1. Show that the function \( g(x) = \sqrt{\frac{x^2 - 9}{x^2}} \) is continuous on its domain and state its domain.

2. The displacement of an object moving in a straight line is given by \( s(t) = 1 + 2t + \frac{1}{4}t^2 \) meters, where \( t \) is measured in seconds. Find the average velocity over the time period \([1, 3]\) and the instantaneous velocity when \( t = 1 \).

3. If \( f(x) = \sqrt{3 - 5x} \), use the definition of the derivative to find \( f'(x) \).

4. Find \( y' \), where \( xe^y = y \sin x \).

5. Find a parabola \( y = ax^2 + bx + c \) that passes through the point \((1, 4)\) and whose tangent lines at \( x = -1 \) and \( x = 5 \) have slopes 6 and -2, respectively.

6. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm\(^3\)/s, how fast is the water level rising when the water is 5 cm deep?

7. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with diameter 50 m.

8. Find the local and absolute extreme values of the function \( f(x) = \sqrt{x^2 + x + 1} \) on the interval \([-2, 1]\).

9. Evaluate the limit \( \lim_{x \to -\infty} (x^2 - x^3)e^{2x} \).

10. Use calculus to sketch the curve \( y = \frac{x}{e^{x^2}} \) (e.g. by finding domain, intercepts, asymptotes, intervals of increase or decrease, local max and min values, concavity, and points of inflection.)

11. Show that the equation \( 3x + 2 \cos x + 5 = 0 \) has exactly one real root.

12. Find the point on the hyperbola \( xy = 8 \) that is closest to the point \((3, 0)\).

13. Evaluate the Riemann sum for \( f(x) = x^2 - x \), \( 0 \leq x \leq 2 \) with four subintervals, taking the sample points to be right endpoints. Draw a sketch to show what the Riemann sum represents.

14. Evaluate the integral \( \int_0^3 |x^2 - 4| dx \).

15. If \( f \) is a continuous function such that
\[
\int_1^x f(t) dt = (x - 1)e^{2x} + \int_1^x e^{-t} f(t) dt
\]
for all \( x \), find an explicit formula for \( f(x) \).