1. Find the determinant of the matrix $A$ using the definition (i.e., patterns and inversions.)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 7 & 5 \end{bmatrix}$$

2. Use the determinant to figure out which values of $k$ make the following matrix invertible.

$$A = \begin{bmatrix} 0 & 1 & k \\ 3 & 2k & 5 \\ 9 & 7 & 5 \end{bmatrix}$$

3. If $A$ is an $n \times n$ matrix and $k$ is an arbitrary constant, what is the relationship between $\det(A)$ and $\det(kA)$?

4. A square matrix is called a permutation matrix if each row and each column contains exactly one 1, and the remaining entries are 0. What are the possible determinants for a permutation matrix? Explain your answer.

5. Use Gauss-Jordan elimination to calculate the determinant of $A$.

$$A = \begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{bmatrix}$$

6. Let $A$ and $B$ be $3 \times 3$ matrices, with $\det A = 4$ and $\det B = -3$. Use properties of determinants to compute:
   a.) $\det AB$
   b.) $\det 5A$
   c.) $\det B^T$
   d.) $\det A^{-1}$
   e.) $\det A^3$
7. Compute the determinant of \( A \) using cofactor expansion.

\[
A = \begin{bmatrix}
1 & -2 & 5 & 2 \\
0 & 0 & 3 & 0 \\
2 & -6 & -7 & 5 \\
5 & 0 & 4 & 4
\end{bmatrix}
\]

8. Use determinants to compute the volume of the parallelepiped with one vertex at the origin and adjacent vertices at \((1, 4, 0), (-2, -5, 2), (-1, 2, -1)\).

9. Use Cramer’s Rule to solve the following linear system.

\[
\begin{align*}
2x_1 + x_2 &= 7 \\
-3x_1 + x_3 &= -3 \\
x_2 + 2x_3 &= -3
\end{align*}
\]

10. Compute the classical adjoint of the given matrix and then give its inverse.

\[
\begin{bmatrix}
1 & 1 & 3 \\
2 & -2 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]