Written Homework 8

Due Tuesday, August 5, 2014

1. Suppose $f$ and $g$ are twice differentiable functions whose second derivatives are never 0.

   (a) If $f$ and $g$ are positive, increasing, concave upward functions on an interval $I$, show that the product function $fg$ is concave upward on $I$.

   (b) Suppose the same conditions as in part (a) except that $f$ is increasing and $g$ is decreasing. Show by giving three examples that $fg$ may be concave upward, concave downward, or linear.

2. Sketch a graph of a function that satisfies all of the given conditions:

   - $f'(1) = f'(-1) = 0$
   - $f'(x) < 0$ if $|x| < 1$
   - $f'(x) > 0$ if $1 < |x| < 2$
   - $f'(x) = -1$ if $|x| > 2$
   - $f''(x) < 0$ if $-2 < x < 0$
   - $f''(x) < 0$ if $-2 < x < 0$
   - inflection point $(0, 1)$

3. Show that

   $$\lim_{x \to \infty} \frac{e^x}{x^n} = \infty$$

   for any positive integer $n$. This shows that the exponential function approaches infinity faster than any power of $x$. 
