Calculus I
Final
Read the directions for each problem, it will be helpful!
Good Luck!

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1. (8 points) Find the equation of the tangent line to $y = xe^x$ at the point $(1, 1)$. 
2. (8 points) Use the definition of the derivative to evaluate $f'(x)$ for $f(x) = \frac{1}{x^2}$
3. (7 points) Evaluate \( \int_{0}^{3} \left( \sqrt{9-x^2} - (x-3) \right) \, dx \)

*Hint: Interpret this as an area. Draw a picture!*
4. (3 points each) The graph of \( f(x) \) is given below.

(a) What is \( \int_{-3}^{1} f(x) \, dx \)?

(b) At what points is \( f \) not continuous?

(c) At what points is \( f \) not differentiable?

(d) What is \( \int_{3}^{4} f^{-1}(y) \, dy \)?
5. (8 points) For what value of $a$ is the following expression true?

\[
\lim_{x \to \infty} \left( \frac{x + a}{x - a} \right)^x = e
\]
6. (9 points) The volume of a cube is increasing at a rate of $10\text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 3cm?
7. (9 points) Find two positive numbers such that sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.
8. (8 points) Calculate

$$\int_{0}^{\pi} \frac{\sin x \cos x + \cos x}{\sin^2 x + 1} \, dx$$
9. (8 points) Write \( \int_{1}^{4} \left( x + \sin(x^2) \right) \, dx \) as a limit of right Riemann sums. DO NOT EVALUATE.
10. (8 points) If $x \tan(\pi x) = \int_0^x f(t)dt$, where $f$ is continuous function, find $f(8)$. 
11. (15 points) Find the below information for the function

\[ f(x) = (x - 2)e^{-x} \]

and use it to sketch its graph on the last page page

(a) Domain

(b) Horizontal and vertical asymptotes (if any)

(c) Zeros of \( f(x) \)
(d) Intervals on which \( f(x) \) is increasing/decreasing

(e) Local maxima and minima

(f) Intervals on which \( f(x) \) is concave up/concave down

(g) Points of inflection
(h) Graph of $f(x)$
Bonus (10 points): For what values of $c$ is there a straight line that intersects the curve

$$y = x^4 + cx^3 + 12x^2 - 5x + 2$$

in four distinct points?