Calculus I
Midterm I

Read the directions for each problem, it will be helpful!
Good Luck!
1. (3 points each) Warm up!

   (a) What does it mean for a function $f(x)$ to be one-to-one?

   (b) What is the definition of a function $f(x)$ being continuous at a number $a$?

   (c) State the Intermediate Value Theorem.

   (d) What is the definition of a vertical asymptote?
2. (5 points each) Calculate the following limits.

(a) \( \lim_{x \to \frac{\pi}{6}} \frac{\cos(x + \frac{\pi}{3})}{3 \tan x} \)

(b) \( \lim_{x \to 1} \frac{1-x^3}{x-1} \)

(c) \( \lim_{x \to -\infty} \left( x + \sqrt{x^2 + 7x} \right) \)

(d) \( \lim_{x \to \pi^-} e^{\cot x} \)
3. (15 points) Define:

\[ f(x) = \frac{e^x - e^2}{2e^x - 4} \]

For \( f(x) \) find:

(a) Vertical asymptotes

(b) Horizontal asymptotes

(c) \( x \)-intercepts

(d) \( y \)-intercepts

(e) Where the function is continuous.
4. (10 points) Find the values of $a$ and $b$ that make $f$ continuous everywhere.

$$f(x) = \begin{cases} 
0, & x < -3 \\
\frac{a}{x} \cos \left( \frac{x}{2} \right) + 3b, & -3 \leq x < -1 \\
4ax^2 - 5bx + 2, & x \geq -1 
\end{cases}$$
5. (8 points) The graph of \( g(x) \) is given below.

Calculate:

(a) \( \lim_{x \to -1} g(x) \)

(b) \( \lim_{x \to -2} g(x) \)

(c) \( \lim_{x \to 1} g(x) \)

(d) \( \lim_{x \to -3} g(x) \)
6. (10 points) Calculate $f'(0)$ where

\[
f(x) = \begin{cases} 
  x^2 \sin \left( \frac{1}{x} \right), & x \neq 0 \\
  0, & x = 0 
\end{cases}
\]
7. (3 points each) Differentiate the following functions

(a) $e^{x \sec x}$

(b) $\log_5(1 + 2x)$

(c) $x^{\sin x}$