1. Write in the form \( a + bi \):
   
   (a) \( e^{-\pi i/4} \); (b) \( e^{1+\pi i} \); (c) \( e^{3+i} \).

2. Find all complex numbers \( z \) such that \( z^4 = -1 \). Write the answer in both polar and cartesian coordinates. How many different solutions are there?

3. Find all complex numbers \( z \) such that \( z^5 = -2 - 2i \). (You can leave your answer in polar form.) How many different solutions are there?

4. Solve the equation \( z^2 + \sqrt{32}iz - 6i = 0 \).

5. Recall from class that for any two complex numbers \( z_1, z_2 \in \mathbb{C} \), we have the triangle inequality:
   
   \[ |z_1 + z_2| \leq |z_1| + |z_2| \]

   (a) Give an example when this inequality is strict; that is, when \( |z_1 + z_2| < |z_1| + |z_2| \).
   (b) When can equality occur?
   (c) Using the triangle inequality and a judicious choice of \( z_1 \) and \( z_2 \), prove the reverse triangle inequality:
   
   \[ |z_1 - z_2| \geq |z_1| - |z_2| \]

6. Write the function \( f(z) \) in the form \( u + iv \):
   
   (a) \( z + iz^2 \); (b) \( 1/z^2 \); (c) \( \overline{z}/z \).

7. Is the function \( \overline{z}/z \) continuous at 0? Why or why not? Is the function \( \overline{z}/z \) analytic where it is defined? Why or why not?

8. Compute the derivatives of the following analytic functions:
   
   (a) \( \frac{iz + 3}{z^2 - (2 + i)z + (4 - 3i)} \); (b) \( e^{z^2} \); (c) \( \frac{1}{e^z + e^{-z}} \).

9. Let \( f(z) \) be a complex function. Is it possible for both \( f(z) \) and \( \overline{f(z)} \) to be analytic? (Hint: if they are both analytic, they both satisfy the Cauchy-Riemann equations.)

10. Determine which of the following complex functions are holomorphic by using the Cauchy-Riemann equations:
    (a) \( f(z) = f(x + iy) = (x^3 - 3xy^2 - x) + i(3x^2y - y^3 - y) \);
    (b) \( f(z) = f(x + iy) = x^2 + iy^2 \);
    (c) \( f(z) = f(x + iy) = 2xy + i(y^2 - x^2) \).
11. Let \( f(z) = x^2 + iy^2 \). Evaluate \( \int_C f(z) \, dz \), where \( C \) is:
(a) the straight line joining 1 to 2 + i;
(b) the curve \((1 + t) + t^2i, 0 \leq t \leq 1\).
Are the results the same? Why might you expect this?

12. Let \( f(z) = -2xy + i(x^2 - y^2) \). Evaluate \( \int_C f(z) \, dz \), where \( C \) is:
(a) the straight line from 0 to 1 + i;
(b) the curve \( t^3 + ti, 0 \leq t \leq 1 \).
Are the results the same? Why might you expect this?

13. Let \( C \) be a circle centered at 4+i of radius 1. Without any calculation, explain why \( \int_C \frac{1}{z} \, dz = 0 \).

14. Let \( C \) be the curve defined parametrically as follows:
\[
z(t) = t(1-t)e^t + \cos(2\pi t^3)i, \quad 0 \leq t \leq 1.
\]
Evaluate the integral \( \int_C e^{z^2} \, dz \). Be sure to explain your reasoning!