From Stewart:
§8.3: 48 (read the section for the definition of centroid—it is the same as the center of mass if $\rho$ is constant)
§8.5: 4, 5, 18, 19
§10.1: 17, 22, 28, 40, 43
§10.2: 34, 41, 69
§10.3: 51, 54, 65

Additional Problems:
1. Consider the same situation as in class—there are two bidders, bidder 1 and bidder 2, who are trying to purchase the same item. The bidders’ values are unknown to the seller and to each other. That is, bidder 1 knows his own valuation $v_1$ of the item, but does not know bidder 2’s valuation $v_2$. These valuations are distributed identically and independently between 0 and 1, with respect to the following probability density function

$$f(x) = \begin{cases} 
3x^2 & 0 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases}$$

Note that before we dealt with the much simpler uniform distribution—here we deal instead with a quadratic density function.

(a). Verify the $f(x)$ is actually a probability density function, i.e. verify that $\int_{-\infty}^{\infty} f(x) \, dx = 1$ and that $f(x) \geq 0$.

(b). Calculate the CDF corresponding to $f$.

Now, let’s do some auction theory. From this part down, everything is extra credit, since it requires a little more econ than I can really expect everyone to be comfortable with. However, even if you don’t know econ, you can still find the right answer by liberal application of common sense.

The seller can decide to sell the item using either the English Auction (open auction) or using the First-Price Sealed Bid Auction. We will show that the two are revenue equivalent for the above distribution.

(c). What are the strategies that bidder 1 and bidder 2, who have valuations $v_1$ and $v_2$ respectively, should adopt if they are playing the English Auction? (Look at what we did in class when we were doing this problem for the uniform distribution. Does what we said here depend on the distribution being uniform?)

(d). Write down bidder 1’s expected profit $\Pi(b_1)$ in the First-Price Sealed Bid Auction if he bids $b_1$, and bidder 2 bids $b_2 = kv_2$ for some $k$. Maximize $\Pi(b_1)$ to find his strategy. What proportion of $v_1$ should bidder 1 bid in order to maximize his expected profits?

(e). What is the expected sale price in the English Auction?

(f). What is the expected sale price in the First-Price Sealed Bid Auction?

Wow! (e). and (f). have the same answer. So there’s something deeper going on in this type of example than just coincidence. In fact, you might postulate that the two auctions are revenue equivalent no matter what distribution $f$ we take. And you’d be right. It goes even further. There’s a broad class of auction types which are all revenue equivalent (being in this class means you satisfy some extremely mild conditions)!