Legendrian links, causality, and the Low conjecture

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Let (X^{m+1}, g) be a globally hyperbolic spacetime with Cauchy surface diffeomorphic to an open subset of \mathbb{R}^m . The Legendrian Low conjecture formulated by Natário and Tod says that two events $x, y \in X$ are causally related if and only if the Legendrian link of spheres $\mathfrak{S}_x, \mathfrak{S}_y$ whose points are light geodesics passing through x and y is non-trivial in the contact manifold of all light geodesics in X. The Low conjecture says that for m = 2 the events x, y are causally related if and only if $\mathfrak{S}_x, \mathfrak{S}_y$ is non-trivial as a topological link. We prove the Low and the Legendrian Low conjectures. We also show that similar statements hold for any globally hyperbolic (X, g) such that the universal cover of its Cauchy surface is diffeomorphic to an open domain of \mathbb{R}^m . An interesting fact, proved in the joint work with Yuli Rudyak, is that a certain weakened version of the Low conjecture is true for all nonrefocussing globally hyperbolic spacetimes. This includes all the cases where a Cauchy surface has infinite fundamental group or is not a closed manifold. (This is based on the joint work with Stefan Nemirovski.)

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