NEW GEOMETRIC-TYPE SOBOLEV INEQUALITIES AND APPLICATIONS TO THE REGULARITY OF MINIMIZERS

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ABSTRACT. We consider the class of stable solutions of semilinear equations \(-\Delta u = f(u)\) in a bounded smooth domain of \(\mathbb{R}^n\) (sometimes convex). This class includes all local minimizers, minimal, and extremal solutions. In dimensions \(n \leq 4\), we establish an a priori \(L^\infty\) bound which holds for every stable solution and every nonlinearity \(f\). In collaboration with M. Sanchón, we prove new weighted Sobolev type inequalities in \(\mathbb{R}^n\), where the weight is a power of the mean curvature of the level sets of the function appearing in the inequality. As an application, we establish new \(L^p\) and \(W^{1,p}\) estimates for stable solutions in all dimensions. Relations with some results for geometric flows of mean curvature type will be discussed.

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