NEW GEOMETRIC-TYPE SOBOLEV INEQUALITIES AND APPLICATIONS TO THE REGULARITY OF MINIMIZERS

XAVIER CABRÉ

ABSTRACT. We consider the class of stable solutions of semilinear equations $-\Delta u = f(u)$ in a bounded smooth domain of $\mathbb{R}^n$ (sometimes convex). This class includes all local minimizers, minimal, and extremal solutions. In dimensions $n \leq 4$, we establish an a priori L^∞ bound which holds for every stable solution and every nonlinearity f. In collaboration with M. Sanchón, we prove new weighted Sobolev type inequalities in $\mathbb{R}^n$, where the weight is a power of the mean curvature of the level sets of the function appearing in the inequality. As an application, we establish new L^p and $W^{1,p}$ estimates for stable solutions in all dimensions. Relations with some results for geometric flows of mean curvature type will be discussed.

ICREA AND UNIVERSITAT POLITÈCNICA DE CATALUNYA, DEPARTAMENT DE MATEMÀTICA APLICADA I, DIAGONAL 647, 08028 BARCELONA, SPAIN
E-mail address: xavier.cabre@upc.edu