In this talk, we discuss the formation of singularities in higher-dimensional Ricci flow without pointwise curvature assumptions. We show that the space of singularity models with bounded entropy and locally bounded energy is orbifold-compact in arbitrary dimensions. In dimension four, a delicate localized Gauss-Bonnet estimate even allows us to drop the assumption on energy in favor of (essentially) an upper bound for the Euler characteristic. We will also see how these results are part of a larger project exploring high curvature regions in 4d Ricci flow. This is all joint work with Reto Mueller.