In this talk I will discuss the *unstable* free boundary problem

$$\Delta u = -\chi_{\{u>0\}} \qquad \text{in } B_1,$$

arising in applications such as solid combustion, composite membranes, climatology and fluid dynamics.

It is known that solutions to the above problem may exhibit singularities —that is points at which the second derivatives of the solution are unbounded— as well as degenerate points. This causes breakdown of by-now classical techniques. I introduce new ideas based on Fourier expansion of the nonlinearity $\chi_{\{u>0\}}$.

The method turns out to have enough momentum to accomplish a complete description of the structure of the singular set in $\mathbb{R}^3$.

A surprising fact in $\mathbb{R}^3$ is that although

$$\frac{u(r/x)}{\sup_{B_1} |u(r/x)|}$$

can converge at singularities to each of the harmonic polynomials

$$xy, \frac{x^2+y^2}{2} - z^2 \text{ and } z^2 - \frac{x^2+y^2}{2},$$

it may *not* converge to any of the non-axially-symmetric harmonic polynomials $\alpha\left((1+\delta)x^2 + (1-\delta)y^2 - 2z^2\right)$ with $\delta \neq 1/2$.

(This is a joint work with John Andersson (Warwick) and Georg. S. Weiss (Düsseldurf).)