Quasiconformal mappings  $u: \Omega \to \Omega'$  between open domains in  $\mathbb{R}^n$ , are  $W^{1,n}$  homeomorphisms whose dilation  $K = |du|/(\det du)^1/n$  is in  $L^{\infty}$ . A classical problem in geometric function theory consists in finding QC minimizers for the dilation within a given homotopy class or with prescribed boundary data. In a joint work with A. Raich we study  $C^2$  extremal quasiconformal mappings in space and establish necessary and sufficient conditions for a 'localized' form of extremality in the spirit of the work of G. Aronsson on absolutely minimizing Lipschitz extensions. We also prove short time existence for smooth solutions of a gradient flow of QC diffeomorphisms associated to the extremal problem.