Drawing out a fascinating connection between Bernstein's problem, on the one hand, and the study of global, bounded and monotone solutions to the semilinear elliptic equation $\Delta u = u^3 - u$ in $\mathbb{R}^n$, on the other, a famous conjecture of De Giorgi states that the level sets of such solutions are hyperplanes, at least in dimension $n \leq 8$. The conjecture was verified for $n \leq 8$ by Savin. Recently, Del Pino, Kowalczyk and Wei constructed a counterexample in dimension $n = 9$, using an intricate fixed point argument. In this talk I would like to discuss the construction of such a counterexample in an appealing free boundary variant of De Giorgi's conjecture. Our approach uses only the elementary means of the method of barriers.