I will discuss joint work with Lionel Levine (Cornell) and Wesley Pegden (NYU). The Abelian sandpile is a diffusion process for distributions of chips on the integer lattice $\mathbb{Z}^d$. The stabilized single-source Abelian sandpile has a distinctive image that solves a discrete elliptic obstacle problem. The continuum limit of this obstacle problem has some curious properties which help explain the fractal-like structure of the stabilized sandpile.