

The counterpart of the Sobolev inequality in the two-dimensional case is the Trudinger-Moser inequality

$$\sup_{u \in H_0^1(\Omega) \|u\|_2=1} \int e^{4\pi u^2} dx < \infty,$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain. The nonlinearity $\int e^{4\pi u^2}$ is, however, sequentially weakly continuous at any nonzero point of $\{\|\nabla u\|_2 < \infty\}$, while the critical nonlinearity $\int |u|^{\frac{2N}{N-2}}$ is not weakly continuous at any point. The lack of weak continuity of the latter can be traced to its invariance with respect to actions of translations and dilations. Furthermore, the functional regains continuity if the weak convergence is replaced by weak convergence "factorized" relatively to translations and dilations. For the Trudinger-Moser case we present two transformation groups playing, respectively, the role translations and dilations, whose actions preserve the gradient norm, and two invariant nonlinearities, stronger than the original Trudinger-Moser nonlinearity, that are not weakly continuous at any point. We give further details concerning the weak convergence for the Sobolev space $H_0^1(\Omega)$ in dimension two, and, more generally, for $W_0^{1,N}(\Omega)$, $\Omega \subset \mathbb{R}^N$.