

Random matrices were introduced by E. Wigner to model the excitation spectra of large nuclei. The central idea is based on the hypothesis that the local statistics of the excitation spectrum for a large complicated system is universal in the sense that it depends only on the symmetry class of the physical system but not on other detailed structures. Dyson Brownian motion is the flow of eigenvalues of random matrices when each matrix element performs independent Brownian motions. In this lecture, we will explain the connection between the universality of random matrices and the approach to local equilibrium of Dyson Brownian motion. This connection has led to a complete solution of the universality conjecture by Wigner, Dyson and Mehta.

The main tools in our approach are an estimate on the flow of entropy in Dyson Brownian motion and a local semicircle law. One key feature of the entropy estimate is an extension of the logarithmic Sobolev inequality to cases not covered by the convexity criterion of Bakry and Emery.