Goldman Sachs

Quantitative Strategies

FEAR & GREED

IN

VOLATILITY MARKETS

~

Emanuel Derman

Quantitative Strategies Group
Goldman Sachs & Co.

Goldman Sachs

Quantitative Strategies

Are There Patterns to Volatility Changes?

- Since 1987, global index options markets are persistently skewed. How do/should volatilities and the skew change as markets move?
- Every description of data involves an articulated or unarticulated model. There are at least three "models" for volatility change:
  - An apocryphal Sticky-Strike Rule, that reflects Greed;
  - An apocryphal Sticky-Delta Rule, that reflects Moderation;
  - A theoretical Implied Tree Model, that reflects Fear.
- Each rule leads to different predictions for valuing & hedging options. Which works best? And why?

- Traders' daily reports are sometimes unreliable. They focus on liquid at-the-money volatility, a moving target, but they own definite strikes.
- Therefore, ignore everyone and look at the data through the prism of models.

- There appear to be several distinct periods ("regimes") in which different rules seem to hold.
- Often, S&P 500 implied volatilities seems to oscillate between the Fear Rule and the Greed Rule.....
- Producing Moderation in the long run, but not the short.

Goldman Sachs

Quantitative Strategies

Contents

Goldman Sachs

Quantitative Strategies

PART I

INTRODUCTION: GLOBAL INDEX IMPLIED VOLATILITIES

Goldman Sachs

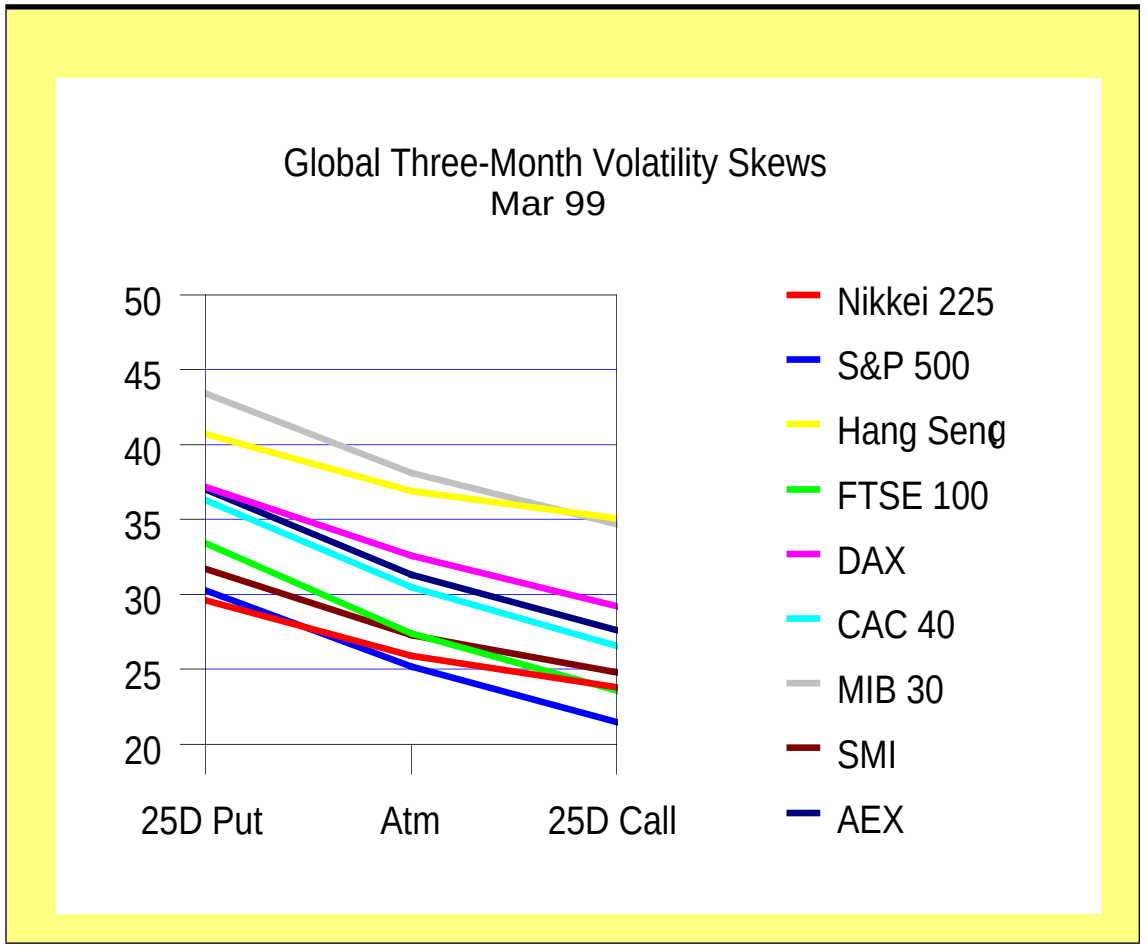
Quantitative Strategies

PART I

INTRODUCTION: GLOBAL INDEX IMPLIED VOLATILITIES

A Persistent Negative Global Skew

A persistent large skew, almost linear, and inconsistent with Black-Scholes.



$$\Sigma(K) = \Sigma_{atm} - b(K - S_0)$$

Goldman Sachs

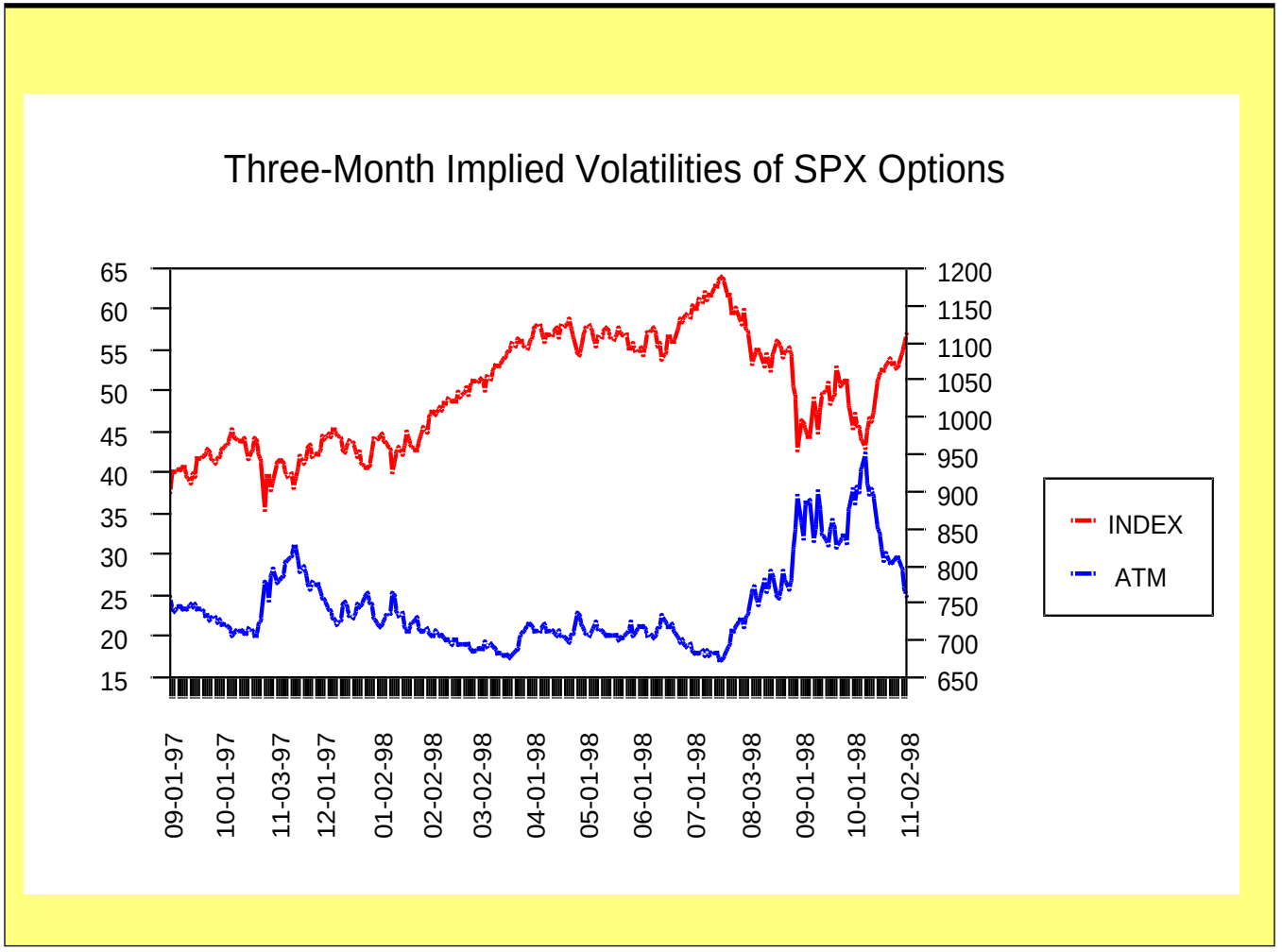
Quantitative Strategies

PART I

INTRODUCTION: GLOBAL INDEX IMPLIED VOLATILITIES

A Negative Correlation with the Index

The S&P 500 index and its at-the-money three-month implied volatility, Sep 1 1997 through Nov 2 1998.

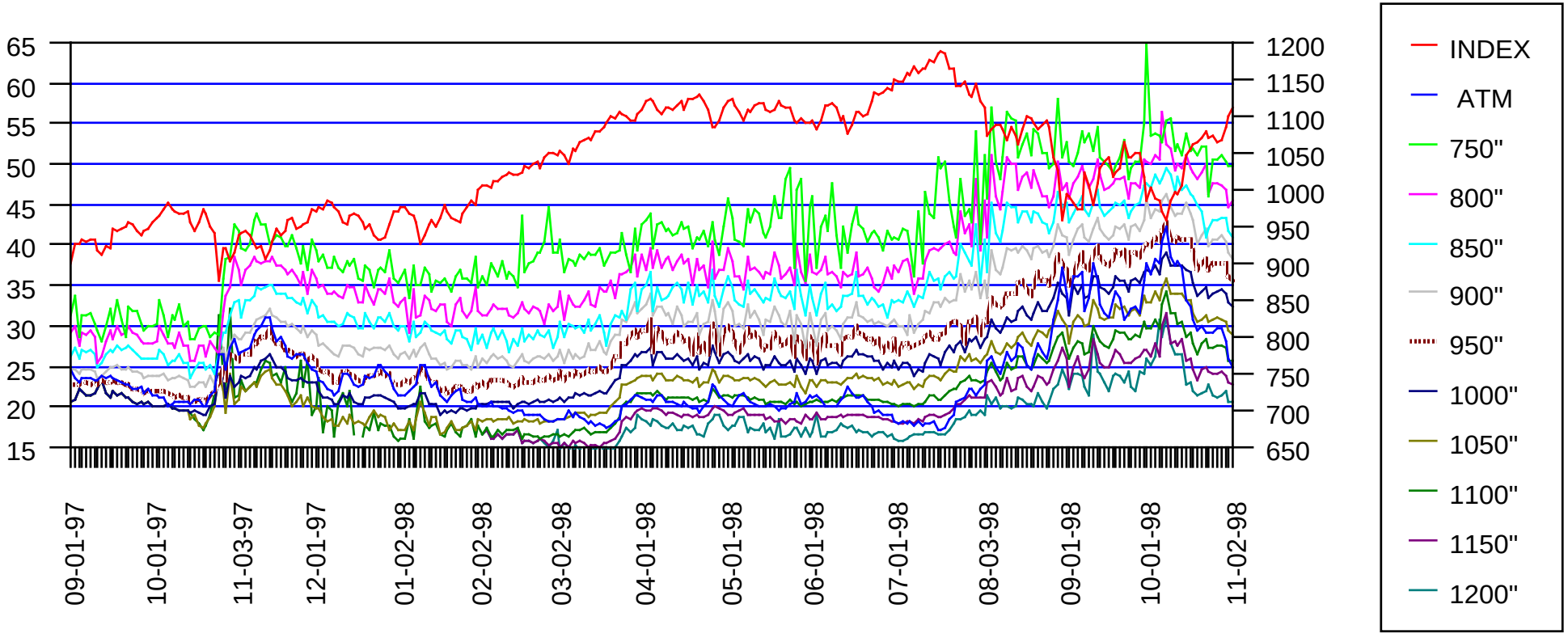


Note - you don't own at-the-money volatility, you own a fixed strike.

Goldman Sachs

Volatility Behavior By Strike Is Complex

Three Month Implied Volatilities of SPX Options



What's going on here?

Goldman Sachs

Quantitative Strategies

PART I

INTRODUCTION: GLOBAL INDEX IMPLIED VOLATILITIES

What's The Future Skew?

We know the current skew $\Sigma(K) = \Sigma_{atm} - b(K - S_0)$.

Hypothetical Implied Volatility of Three-Month SPX Options

Index	103	102	101	100	99	98	97
Strike							
103				17			
102		?		18		?	
101				19			
100		?		20		?	
99				21			
98		?		22		?	
97				23			

- What will happen when the index moves?
- What's the S-dependence in $\Sigma(S,K)$?
- Distinguish carefully between $\Sigma(S,K)$ and $\Sigma_{atm}(S) = \Sigma(S,S)$.

Goldman Sachs

Quantitative Strategies

PART II

GREED
(STICKY STRIKE)

Goldman Sachs

Quantitative Strategies

PART II

GREED (STICKY STRIKE)

Complacency or Greed: Sticky Strike "Model"

The simplest & most convenient model for changing the implied volatility of an option as the index moves is not to change it at all. This is the or complacency model, or "sticky strike," the closest thing to Black-Scholes. It's also the lazy-trader model.

"STICKY STRIKE" $\Sigma(S, K) \equiv \Sigma(K) = \Sigma_{atm} - b(K - S_0)$

Characteristics

- Fixed-strike volatility is independent of S.
- Therefore, because of the negative skew, at-the-money volatility falls with rising S.
- $\Delta = \Delta_{BS}$.

In a rising market, you can think of this model as representing Irrational Exuberance or Greed:

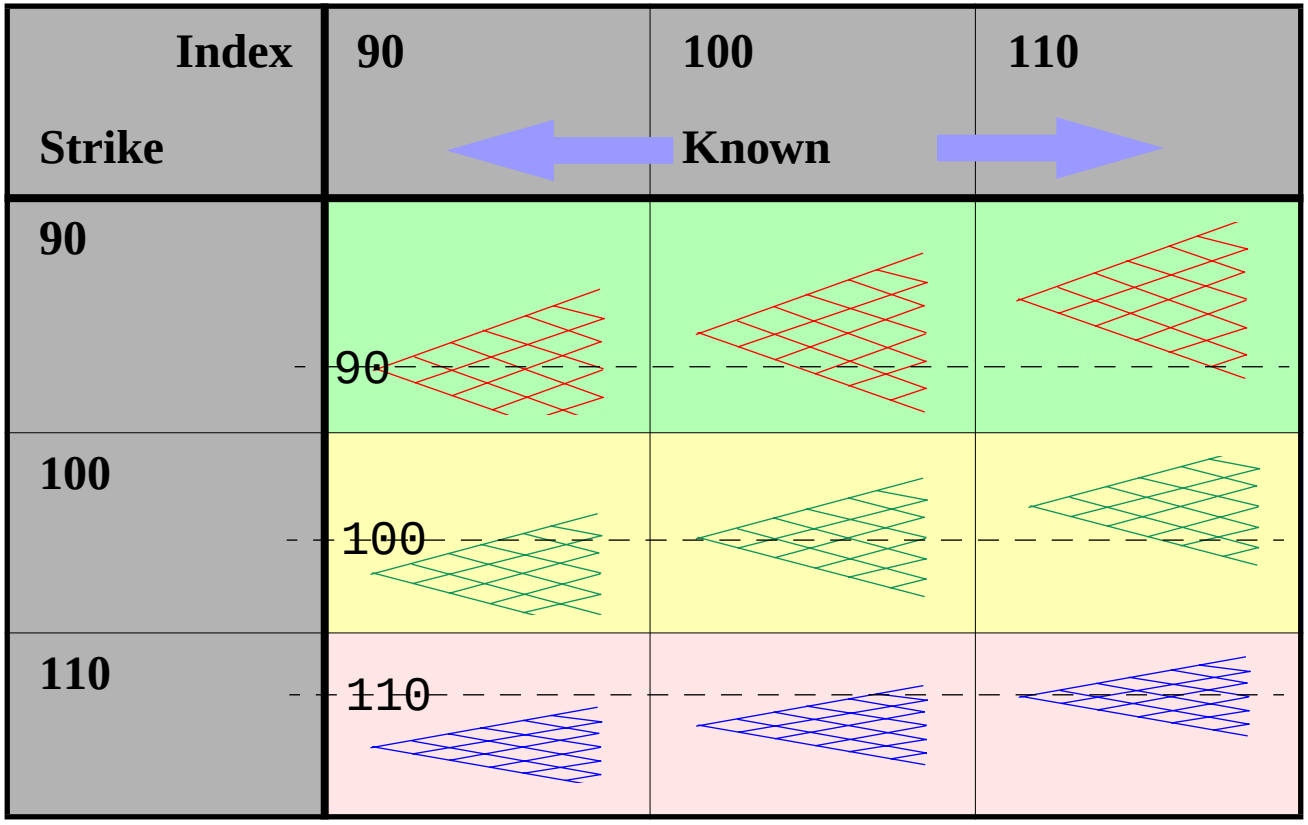
At-the-money options are the most liquid.

When the market rises, at-the-money volatility falls, and you are selling the most liquid options more and more cheaply, as though you need never worry about future index declines.

Goldman Sachs

Quantitative Strategies

How Options Trees Evolve In The Sticky Strike Model



- Fixed-strike volatility is independent of S.
- Therefore, because of the negative skew, at-the-money volatility falls with rising S.
- $\Delta = \Delta_{BS}$.

PART II

GREED (STICKY STRIKE)

PART III

MODERATION (STICKY DELTA)

Goldman Sachs

Quantitative Strategies

PART III

MODERATION (STICKY DELTA)

Rational Moderation

At-the-money volatility is the rational estimate for the future cost of replicating liquid options issued now. On average, over the long run, at-the-money volatility should be independent of index level.

If you have no special expectations about the future, you should keep at-the-money volatility unchanged.

Given the negative skew, as the index rises, you need to raise every strike's volatility to keep at-the-money volatility unchanged.

Traders refer to this as the Sticky Moneyness or Sticky Delta Model.

"STICKY DELTA": $\Sigma = \Sigma(K/S) = \Sigma_{atm} - b(K - S)$

Characteristics

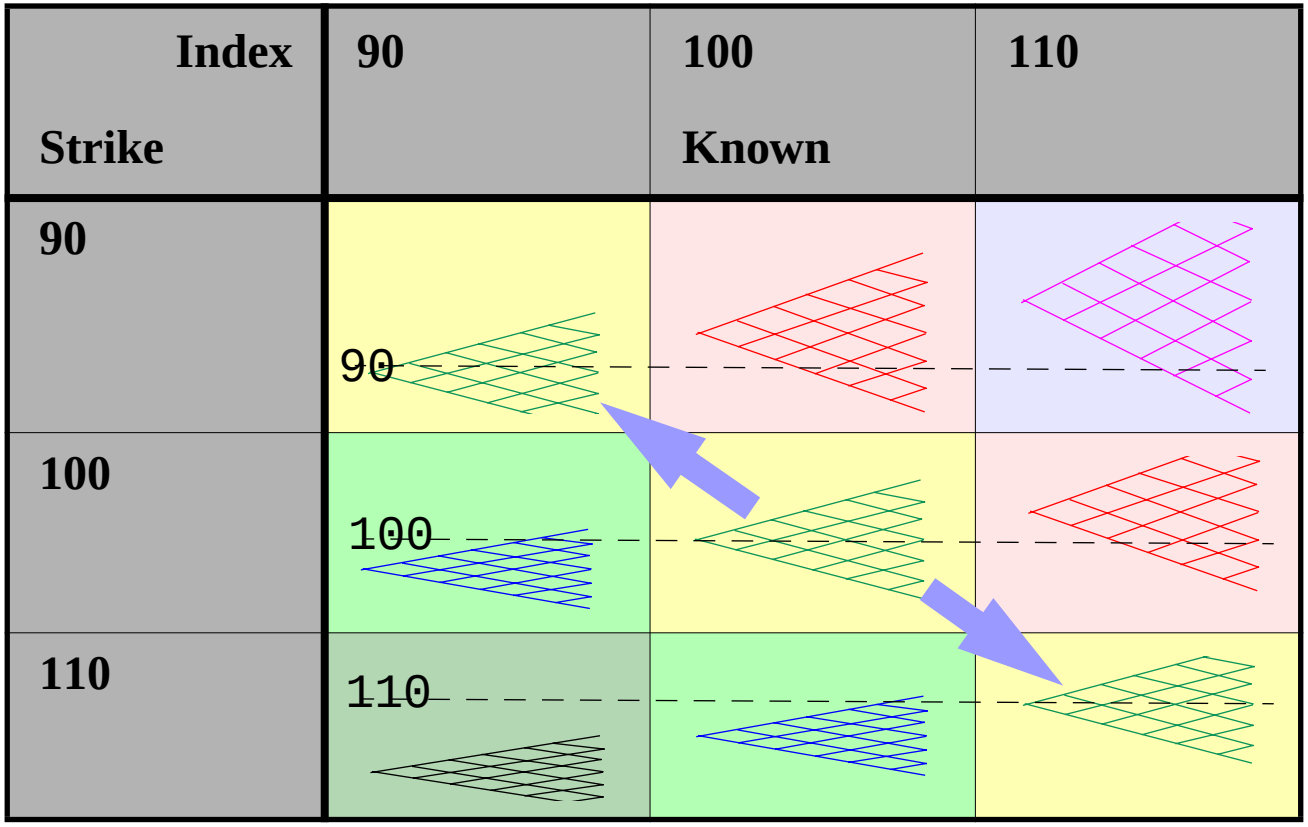
- Atm vol is independent of S.
- Fixed-strike vol increases with S.
- $\Delta > \Delta_{BS.}$

Goldman Sachs

Quantitative Strategies

How Options Trees Evolve In The Sticky Delta Model

.



- Atm vol is independent of S.
- Fixed-strike vol increases with S.
- $\Delta > \Delta_{BS}$.

PART III

MODERATION (STICKY DELTA)

Goldman Sachs

Quantitative Strategies

PART IV

FEAR
(STICKY IMPLIED TREE)

Goldman Sachs

Quantitative Strategies

PART IV

FEAR (STICKY IMPLIED TREE)

Why The Skew? Fear of Index Declines!

The skew represent the premium for the fear of a downward market move and an increase in realized and implied volatility.

Relation between the current skew and the expected future volatility.

Strike	Implied Volatility (%)
100	20%
99	21%
98	22%
97	23%

You can deduce the local volatility at different market levels by treating the implied volatility as an average over local (future at-the-money) volatilities.

Index Level	Local volatility (%)
100	20%
99	22%
98	24%
97	26%

These local volatilities are the future at-the-money volatilities feared to occur in a decline. Note that local volatilities increase twice as fast with index changes as implieds increase with strike.

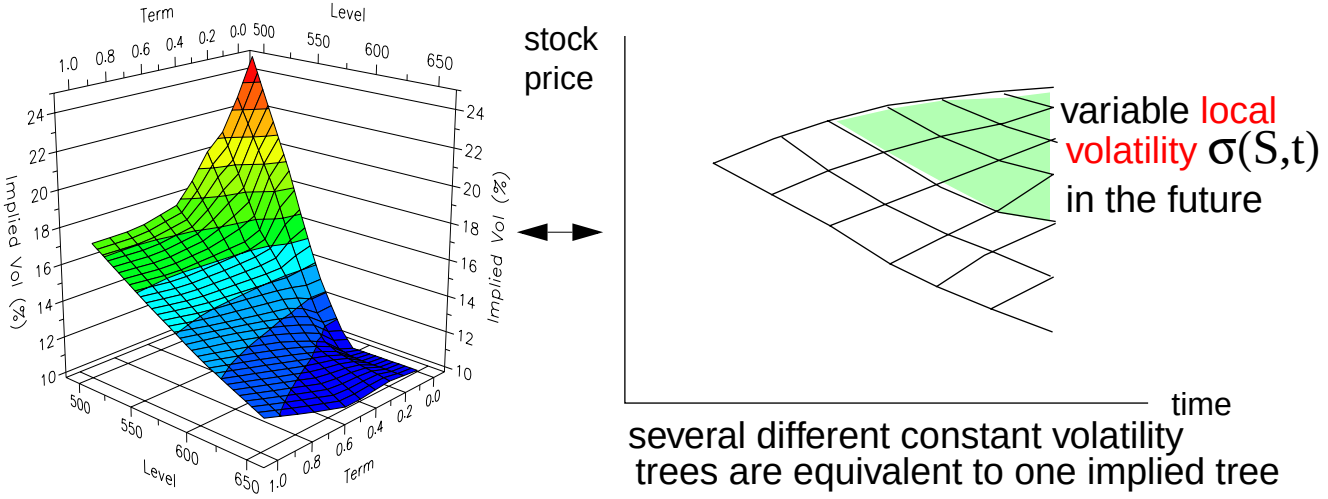
Goldman Sachs

Quantitative Strategies

Sticky Implied Tree Extracts Local Volatilities

There is one market-consistent tree - the implied tree - whose expectations of future volatilities match all current options prices and the skew. In this view, the skew is attributable to an expectation of higher volatility as the market moves (jumps?) down.

You can use this tree to price all options consistently off future implied local volatilities. This is similar to pricing all off-the-run bonds off current forwards.



When the index moves, to find the new skew, you roll along the local vols. This is similar to rolling along the forward curve to get future yields as time passes.

STICKY IMPLIED TREE: $\Sigma(K,S) = \Sigma_{atm} - b(K+S)$

PART IV

FEAR (STICKY IMPLIED TREE)

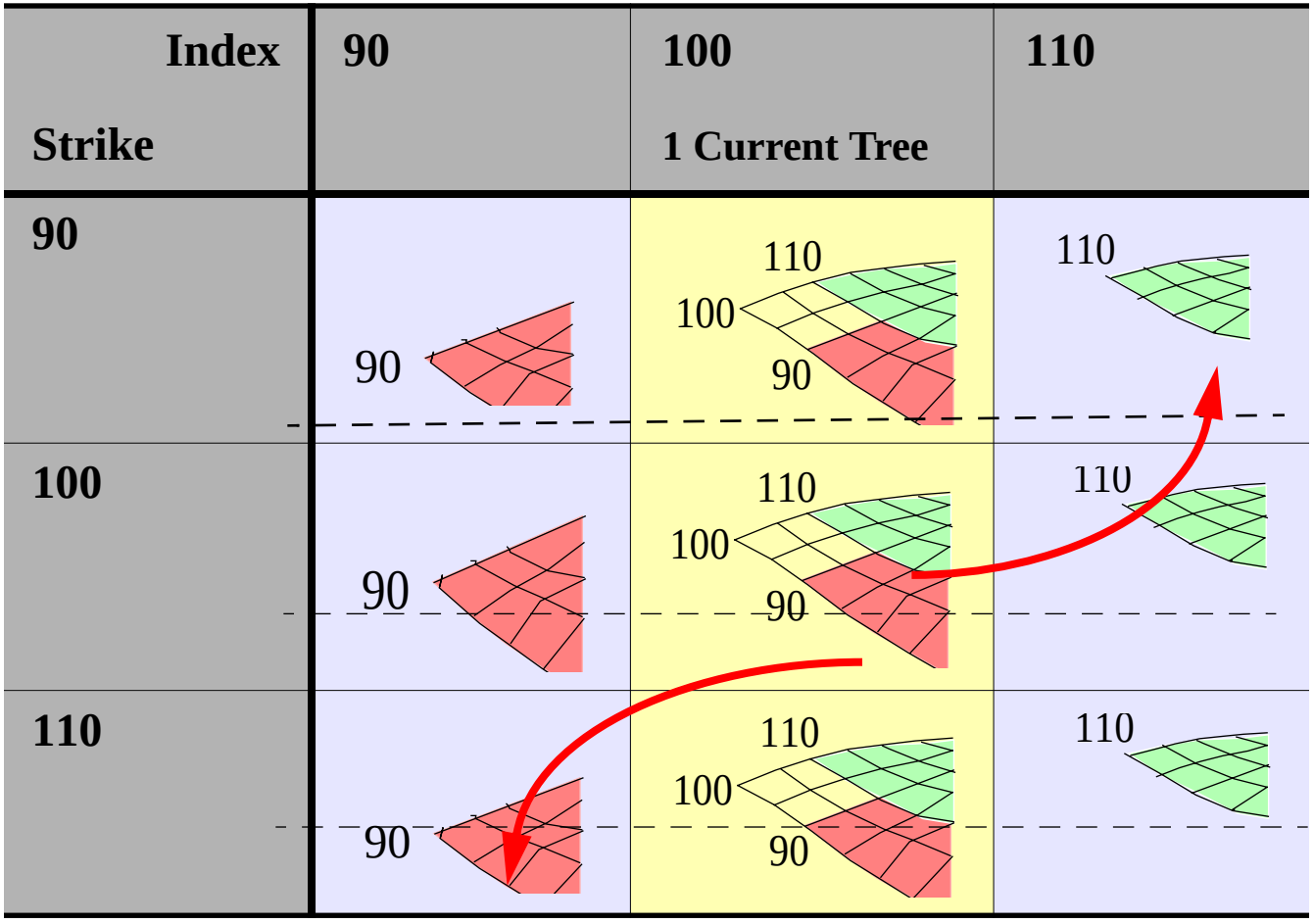
Goldman Sachs

Quantitative Strategies

PART IV

FEAR (STICKY IMPLIED TREE)

How Options Trees Evolve In The Sticky Implied Tree Model



- Fixed-strike volatility decreases as K or S increases.
- Atm vol falls twice as rapidly as skew.
- $\Delta < \Delta_{BS}$.

Goldman Sachs

Quantitative Strategies

PART V

MODEL SUMMARY

Goldman Sachs

Quantitative Strategies

The Properties of the Models

Stickiness Model	Equation for $\Sigma(S, K)$	Behavior of		Delta
		Fixed-strike Option Volatility	At-the-money Option Volatility	
Strike	$\Sigma_{atm}(t) - b(t)(K - S_0)$	independent of index level	decreases as index level increases	$= \Delta_{BS}$
Delta	$\Sigma_{atm}(t) - b(t)(K - S)$	increases as index level increases	independent of index level	$> \Delta_{BS}$
Implied tree	$\Sigma_{atm}(t) - b(t)(K + S)$	decreases as index level increases	decreases twice as rapidly as index level increases	$< \Delta_{BS}$

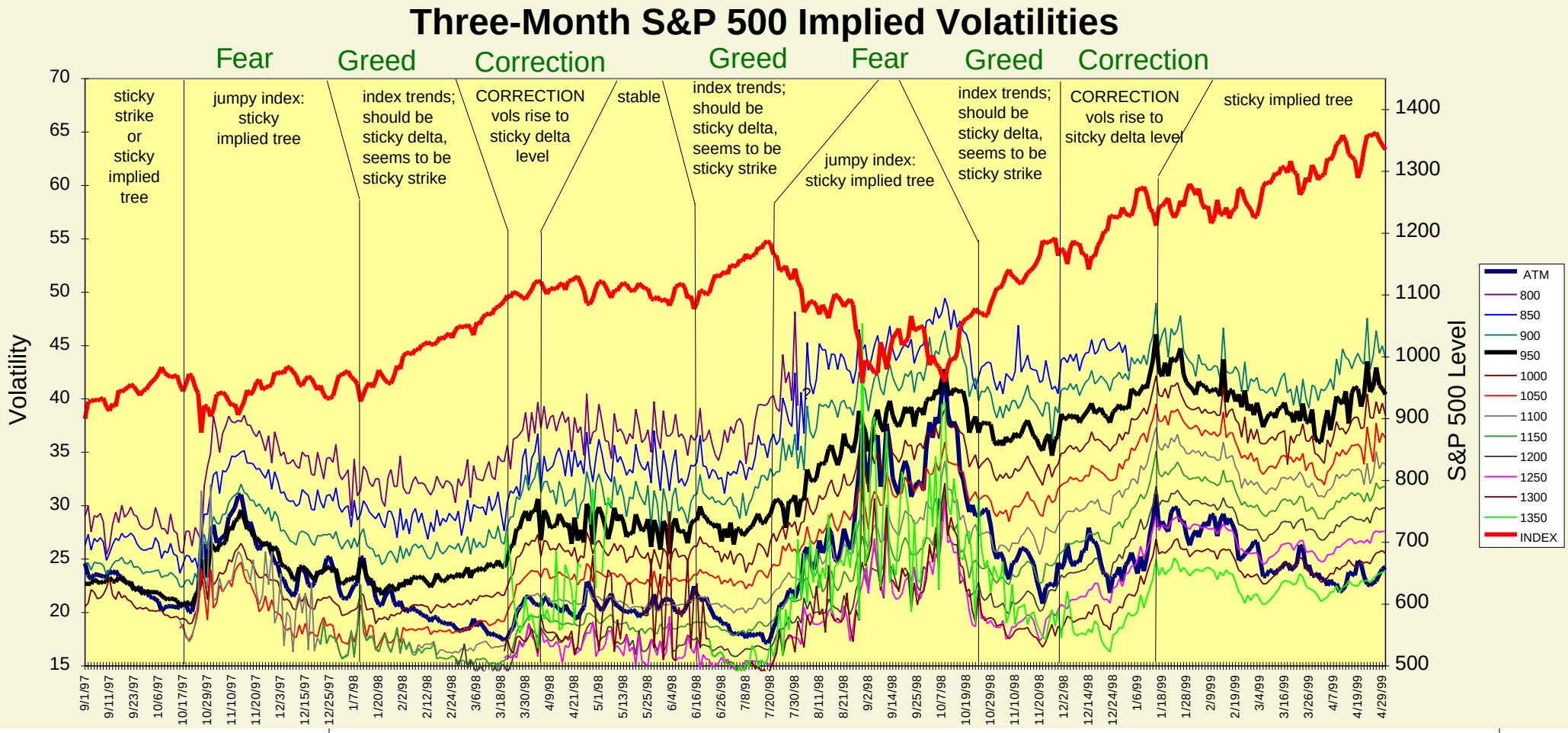
Goldman Sachs

Quantitative Strategies

PART VI

WHAT REALLY HAPPENS: MODEL REGIMES

Which Model Reigns in Which Regime?



Goldman Sachs

Quantitative Strategies

PART VI

WHAT REALLY HAPPENS: MODEL REGIMES

Conclusions

Sticky strike (complacency)
Sticky delta (moderation)
Sticky implied tree (fear)

are intuitively useful ways of thinking about variations in implied volatility that sometimes correspond to modes of market behavior.

- When times are good, and the index keeps rising, the options market keeps every strike's volatility roughly fixed, and so the pendulum of at-the-money volatility drops.

- When times get bad, and the index jumps down a few percentage points, the market has to compensate for having let at-the-money volatility drop too far. The pendulum reverses, and moves at-the-money volatility up at twice the rate as the index collapses.

- On average, over the long haul, the pendulum oscillations between sticky-strike Greed and sticky-implied-tree Fear average out to sticky-delta Moderation.

Will these conjectured regimes extend through time and across markets?

Is there a model of stochastic volatility that encompasses this?

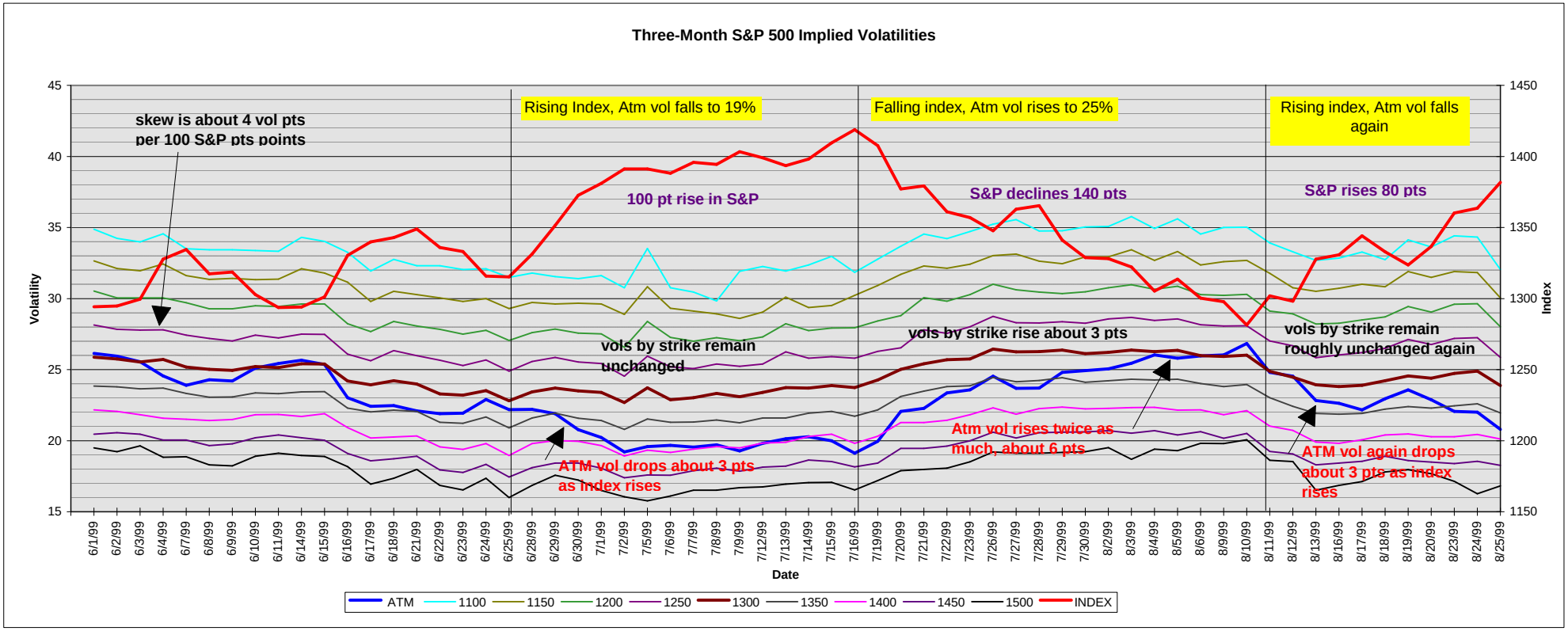
Goldman Sachs

Quantitative Strategies

Recent Update: July-August '99

PART VI

WHAT REALLY HAPPENS: MODEL REGIMES


Three-Month S&P 500 Implied Volatilities
Rising Index, Atm vol falls to 19%
Falling index, Atm vol rises to 25%
Rising index, Atm vol falls again
skew is about 4 vol pts per 100 S&P pts points
100 pt rise in S&P
S&P declines 140 pts
S&P rises 80 pts
vols by strike remain unchanged
vols by strike rise about 3 pts
vols by strike remain roughly unchanged again
Atm vol rises twice as much, about 6 pts
ATM vol drops about 3 pts as index rises
ATM vol again drops about 3 pts as index rises
Volatility
Index
Date
ATM
1100
1150
1200
1250
1300
1350
1400
1450
1500
INDEX