DIRICHLET'S THEOREM ON PRIMES IN ARITHMETIC PROGRESSIONS

RAHUL KRISHNA

In 1837, Dirichlet proved his striking theorem on primes in arithmetic progressions: for every pair of integers a, q with $(a, q) = 1$, the sum

$$\sum_{\substack{p \equiv a \ (q) \\ p \text{ prime}}} \frac{1}{p}$$

diverges–as a corollary, there are infinitely many primes $p \equiv a \bmod q$. In this talk I will introduce the central objects in the proof, the Dirichlet L-functions $L(s, \chi)$, and use them to prove the theorem. We will loosely follow Dirichlet's original argument, but substitute in some more modern techniques to try and keep things as elementary as possible. Prerequisites: some familiarity with abelian groups and a little bit of complex analysis.