1. Suppose $0 \leq x \leq 1$. Let $f(x) = \sqrt{1 - x^2} + \frac{2}{\pi} \sin^{-1}(x)$. Show that $f(x) \geq 1$.
   \textit{Hint:} I do not want you to use a calculator. You should use the techniques of Sections 4.1 and 4.3 of Stewart.

2. A woman at a point $A$ on the shore of a circular lake with radius 1 wants to arrive at the point $C$ diametrically opposite $A$ on the other side of the lake in the shortest possible amount of time. She can either walk around the lake, row straight across the lake, or row in a straight line to a point $B$ on shore and walk the rest of the way. (See the figure following Problem 48 in Section 4.7 of Stewart. However, note that this problem is different.) She can walk at a rate of $w$ m/s and row at a rate of $r$ m/s. The goal of this problem is to show that it is always faster to walk the whole way or row the whole way. You are welcome to approach this your own way, but below is an outline of one approach.
   
   (a) Express the time $T$ of travel as a function $T(\theta)$ of $\theta$, the angle shown in the figure.
   \textit{Hint:} This will involve some circle geometry.

   (b) Thus, we want to minimize $T$ over the interval $0 \leq \theta \leq \frac{\pi}{2}$ and show that $\theta = 0$ or $\theta = \frac{\pi}{2}$ is always the absolute minimum. To do this we follow the procedure of Section 4.1. Show that if $r \geq w$, then $T(\theta)$ has no critical points in the interval $0 \leq \theta \leq \frac{\pi}{2}$. Show that if $w \geq r$, then $T(\theta)$ has one critical point in the interval $0 \leq \theta \leq \frac{\pi}{2}$.

   (c) Suppose $w \geq r$ and let $\theta_0$ denote the critical point from part (b). We need to show that $T(\theta_0) \leq T(0)$ or $T(\theta_0) \leq T(\frac{\pi}{2})$. To do this, break into two cases:
   
   (i) If $T(\frac{\pi}{2}) \leq T(0)$, show that $T(\theta_0) \geq T(\frac{\pi}{2})$.
   (ii) If $T(0) \leq T(\frac{\pi}{2})$, show that $T(\theta_0) \geq T(0)$.
   \textit{Hint:} Let $x = \frac{\theta}{\frac{\pi}{2}}$, so $0 \leq x \leq 1$, and express $T(\theta_0)$ as a function of $x$. Then use Problem 1 to solve parts (i) and (ii). Note that this is not the only way to approach this part.