An open interval in $\mathbb{R}$ is of the form $(a, b)$, where $-\infty \leq a < b \leq +\infty$.

1. Let $X$, $Y$, $Z$ be the vector fields defined on $\mathbb{R}^3$ by
   
   \[ X = z \frac{\partial}{\partial y} - y \frac{\partial}{\partial z}, \quad Y = x \frac{\partial}{\partial z} - z \frac{\partial}{\partial x}, \quad Z = y \frac{\partial}{\partial x} - x \frac{\partial}{\partial y}. \]

   Compute the flow of the vector field $aX + bY + cZ$ where $a, b, c \in \mathbb{R}$.

2. Let $X, Y, Z$ be smooth vector fields on a smooth manifold. Verify the Jacobi identity:
   \[ [X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0. \]

3. Let $X$ be a smooth vector field on a smooth manifold $M$, and let $\gamma : I \to M$ be a nonconstant integral curve of $X$, where $I$ is an open interval in $\mathbb{R}$. Prove the following statements.
   (a) $\gamma$ is an immersion.
   (b) If $\gamma$ is not injective, then there exists a smooth embedding $i : S^1 \to M$ such that $i(S^1) = \gamma(I)$.

4. Let $X$ be the vector field on $\mathbb{R}$ defined by $X(x) = x^2 \frac{\partial}{\partial x}$. Given $x \in \mathbb{R}$, let $\phi_x : I_x \to \mathbb{R}$ be the unique integral curve of $X$ such that $\phi_x(0) = x$, where $I_x$ is an open interval containing $0$, and $\phi_x$ cannot be extended to a larger open interval containing $I_x$. Find $\phi_x$ and $I_x$ for all $x \in \mathbb{R}$. 