LIPSCHITZ LECTURE 1 ACCOMPANYING EXERCISES

IVAN CORWIN

Abstract. Feel free to come by office hours Wednesday and Thursday 3 - 5 p.m. in room 3-040.

(1) Recall h_k, e_k, p_k are the homogeneous, elementary and power-sum symmetric functions in variables $x_1, x_2, \ldots$. Let $H(z) = \sum_{k=0}^{\infty} h_k z^k$, $E(z) = \sum_{k=0}^{\infty} e_k z^k$ and $P(z) = \sum_{k=1}^{\infty} p_k z^{k-1}$. Prove that these formal power series satisfy
  (a) $H(z) = \prod_i \frac{1}{1-x_i z}$,
  (b) $E(z) = \prod_i (1 + x_i z)$,
  (c) $P(z) = \frac{d}{dz} \sum_i \log\left(\frac{1}{1-x_i z}\right)$,
  (d) $H(z) = \frac{1}{E(-z)} = \exp\left\{\sum_{k=1}^{\infty} \frac{p_k z^k}{k}\right\}$.

(2) The monomial symmetric functions m_λ and the power-sum symmetric functions $p_\lambda = p_{\lambda_1} p_{\lambda_2} \cdots$ form linear bases of Λ. The space spanned by these bases is Λ which is naturally graded with respect to the degree. Work out the transition matrix between these two linear bases when restricted to symmetric functions of degree ≤ 3. (Hint: The matrix has block form based on the degree. For example when degree is 2, the block should correspond to the transition matrix between $(m_{(2)}, m_{(1,1)}$ and $p_{(2)}, p_{(1,1)})$.

(3) The symmetric functions $\{e_k\}_{k\geq 0}$, $\{h_k\}_{k\geq 0}$ and $\{p_k\}_{k\geq 0}$ all constitute algebraically independent sets of generators of Λ. This means that every element in Λ can be written in terms of these generating sets via the $(+, \times)$ operators.
  (a) Write p_2 in terms of h_k's.
  (b) Write e_3 in terms of p_k's.
  (c) Write h_3 in terms of e_k's.