(1) Prove that
\[
\sum_{\sigma \in S_k} \prod_{1 \leq B < A \leq k} \frac{z_{\sigma(A)} - qz_{\sigma(B)}}{z_{\sigma(A)} - z_{\sigma(B)}} = k_q! := \frac{(1 - q)(1 - q^2) \cdots (1 - q^k)}{(1 - q)(1 - q) \cdots (1 - q)}.
\]

(2) Prove that \( k_q! \to k! \) as \( q \to 1 \).

(3) Hahn (1949) introduced two \( q \)-deformed exponential functions
\[
e_q(x) = \frac{1}{(1 - q)x; q)_\infty}, \quad E_q(x) = (1 - (1 - q)x; q)_\infty.
\]
(Recall \( (a; q)_\infty = \prod_{i \geq 0} (1 - q^i a) \)) Prove that for any fixed \( x \), as \( q \to 1 \), \( e_q(x), E_q(x) \to e^x \).

(4) The \( q \)-Laplace transform is defined for any function \( f \in \ell^1(\mathbb{Z}_{\geq 0}) \) as
\[
\hat{f}^q(z) := \sum_{n \geq 0} f(n) \frac{z^n}{(zq^n; q)_\infty},
\]
where \( z \in \mathbb{C} \setminus \{q^{-M}\}_{M \geq 0} \). Prove the following inversion formula:
\[
f(n) = -q^n \frac{1}{2\pi i} \int_{C_n} (q^{n+1} z; q)_\infty \hat{f}^q(z) dz
\]
where \( C_n \) is any positively oriented complex contour which encircles only the poles \( z = q^{-M} \) for \( 0 \leq M \leq n \). (Hint: Start by computing the residues of the integrand above at these pole in terms of the values of \( f(n) \).)

(5) Prove the duality statement between \( q \)-TASEP and \( q \)-TAZRP. Recall that
\[
(L^q-TASEP f)(\bar{x}) = \sum_{i=1}^N (1 - q^{x_{i-1} - x_i - 1}) [f(\bar{x}_i^+) - f(\bar{x})]
\]
\[
(L^q-TAZRP h)(\bar{y}) = \sum_{i=1}^N (1 - q^{y_i}) [h(\bar{y}^+; i - 1) - h(\bar{y})]
\]
\[
H(\bar{x}; \bar{y}) = \prod_{i=0}^N q^{(x_{i+1})y_i}.
\]
Then prove the identity
\[
L^q-TASEP H(\bar{x}; \bar{y}) = L^q-TAZRP H(\bar{x}; \bar{y})
\]
where on the left the operator acts on \( \bar{x} \) and on the right it acts on \( \bar{y} \).