**Proposition** A for a morphism $X_0 \to Y_0$, $F$ pullback functor $\text{Qcoh}(Y_0) \to \text{Qcoh}(X_0)$ which is an equivalence if $F$ is Morita, and natural trans. gives iso. of functors.

The cleanest way to think of this is given a diagram of schemes, what is a quasi-coherent sheaf on that diagram (Cartesian)?

![Diagram](image)

$$\begin{array}{ccc}
\mathcal{E} & \to & \mathcal{F} \\
\downarrow & & \downarrow \\
\mathcal{E}' & \to & \mathcal{F}'
\end{array}$$

\text{arrow } \mathcal{E} \to \mathcal{F} \text{ is an isomorphism.}

Claim: A quasicoherent sheaf is the same as a Cartesian quasicoherent sheaf on the diagram

$$\begin{array}{ccc}
X_2 & \to & X_1 & \to & X_0
\end{array}$$

**Proof of prop:**

Step 1) Ranal around for $F \circ F$ man.
Step 1) Banal groupoid for Fppf map

- Can reduce to case of affine Fppf maps, statement about ring maps

2) For general groupoids, look at

\[
\begin{array}{ccc}
X_0 & \rightarrow & Y_0 \\
\downarrow & & \downarrow \\
W_0 & \rightarrow & Y_0 \\
\end{array}
\]

3) Use the fact that

\[
\text{QCoh}(X_0) \rightarrow \text{QCoh}(W_0) \rightarrow \text{QCoh}(Y_0)
\]

Thus we think of \(\text{QCoh}(X_0)\) as indep. of presentation.

Already, we secretly used the notion of Fibrant categories in definition existence.
Already we secretly used the notion of fibered categories: \textit{definition: existence of Cartesian arrows.}

We saw the example of $\mathcal{O}_{\text{Coh}} = \mathcal{O}(X, \xi)^2$.

Think of this as a functor from schemes to categories $\mathcal{O}_{\text{Coh}}(U) := p^{-1}(U) \circ \mathcal{O}_{\text{Coh}}$.

Given a diagram of schemes, a quasi-coherent sheaf on $\mathcal{O}_{\text{Coh}}$ on what diagram is a Cartesian section $\xi \rightarrow \mathcal{O}_{\text{Coh}}$

This leads to the correct notion of a stack (modeled after the stack of q-coh. sheaves).
**Def.** A stack is a category fibered in groupoids such that a local groupoid associated to an etale cover \( U \to X \) of schemes,

\[
\mathcal{F}(X) \to \mathcal{F}(U_0)
\]

is an equivalence.

4.1.3. Fibered categories with descent.

**Definition 4.6.** Let \( \mathcal{F} \to C \) be a fibered category on a site \( C \).

(i) \( \mathcal{F} \) is a prestack over \( C \) if for each covering \( \{U_i \to U\} \) in \( C \), the functor \( \mathcal{F}(U) \to \mathcal{F}(\{U_i \to U\}) \) is fully faithful.

(ii) \( \mathcal{F} \) is a stack over \( C \) if for each covering \( \{U_i \to U\} \) in \( C \), the functor \( \mathcal{F}(U) \to \mathcal{F}(\{U_i \to U\}) \) is an equivalence of categories.

**Ex.** the stack \( \text{Qcoh}^{\text{art}} \)

**Ex.** Stack of \( G \)-bundles

**Ex.** Quotient stacks

these are algebraic.

To define this, some notions

1) base preserving functors
2) 2-fiber product
2) 2-fiber product
3) 2-Yoneda Lemma

2-YONEDA LEMMA. The two functors above define an equivalence of categories
\[ \text{Hom}_C((C/X), F) \cong F(X). \]

4) definition of representable maps