Name: **First Midterm**

Write your name on this paper. Explain all your answers. Use common sense. Do not use calculators/book/notes. There are six parts all equally weighted. Here is the first one:

1. Find the inverse of the function

(a) $f(x) = 3x + 12$.

(b) $g(t) = 13e^t$.

2. Miscellaneous questions:

(a) Why is $f(x) = \sqrt{x}$ increasing on its domain $[0, \infty)$?

(b) Is $g(x) = \sqrt{-x - 1}$ increasing?

(c) What is the domain of the function

$$h(x) = \frac{1}{x+1} + \frac{1}{x+13} \ ?$$

3. Evaluate the following limits.

(a)

$$\lim_{x\to\infty} \frac{x^2+x+1}{3x^2-2x+1}$$

(b)

$$\lim_{x\to 0} \frac{x+\sqrt{1+x}-\sqrt{1-x}}{x}$$

(c)

$$\lim_{x\to 0} e^{sin(x)}$$

4. Consider the function defined by the rule

$$f(x) = \begin{cases} -x & if \quad x < -10 \\ 10 & if \quad -10 \leq x \leq -5 \\ x^2 & if \quad -5 < x < 0 \\ 0 & if \quad 0 \leq x \end{cases}$$

(a) Is f continuous at $a = -10$?
(b) Is f continuous at $a = -5$?
(c) Is f differentiable at $a = 0$?

5. Give your own definition of the following notions. Please try to be concise, and make the definitions mean the same thing as the definitions that we discussed in lectures or that are in the book.

(a) The *composition* $f \circ g$ of f and g.

(b) The *limit* $\lim_{x \to a} f(x)$. (You may either recall the precise definition or rephrase the intuitive definition that came first.)

(c) The notion of *continuity* of a function f at a.

(d) The notion of *being differentiable at* a of a function f.

6. Inequalities.

(a) Find a $\delta > 0$ such that for all x with $0 < |x| < \delta$ we have

$$\left| \frac{x^2}{|x| + 4x} \right| < \frac{1}{100}.$$

(b) What is your guess for some $\delta > 0$ which works in (a) if we replace the quantity $\frac{1}{100}$ in the formula above by ϵ for some positive number ϵ? No explanation necessary for this part.

(c) Find a $\delta > 0$ such that for all x with $0 < |x| < \delta$ we have

$$\left| x + \frac{1}{1000} \right| < \frac{1}{200}.$$

(d) Is there some $\delta > 0$ which works in (c) if we replace the quantity $\frac{1}{200}$ in the formula just above by $\frac{1}{10000}$? No explanation necessary for this part.