

Name:

Second Midterm

Write your name on this paper. Explain all your answers. Use common sense. Do not use calculators/book/notes. There are six parts all equally weighted. Here is the first one:

1. Find the derivative of the following functions.

(a) $f(x) = 100x^2 + e^{x^2+x}$.

(b) $g(t) = \frac{1}{1+\sin^2(t)}$.

2. Consider the equation

$$2^x + 2^y + 2^{2y} = 8$$

- (a) Find $\frac{dy}{dx}$.
- (b) Find the tangent line to the curve at the point $(1, 1)$.

3. Concavity.

(a) What is the definition of an inflection point?

(b) Find the inflection points of the graph of $f(x) = 2x^6 - 5x^4 + 20x - 10$.

4. Consider the function $f(x) = 2|x - 1| - 2x^2 + 3x$ on the interval $[-1, 2]$.
- (a) Find the critical points of f .
 - (b) Find the local minima and maxima.
 - (c) Find the absolute minima and maxima.
 - (d) Sketch the graph of f using the data you found above.

5. Rolle's theorem.

- (a) State Rolle's theorem.
- (b) Suppose that f is a differentiable function on $(-1, 4)$ and suppose that $f(0) = f(1) = f(2) = f(3) = 0$. Is it possible that f' has fewer than 3 zeros?
- (c) Give an example of a function f as in (b) whose derivative has exactly 3 zeros.
No explanation necessary for this part.

6. Compute the following limits.

(a)

$$\lim_{x \rightarrow \infty} \frac{x + \ln(x)}{x - 1}$$

(b)

$$\lim_{x \rightarrow 0} \frac{\cos(x) - \cos(2x)}{x}$$