Sheaves

1. Carefully prove that a map of sheaves of sets is an epimorphisms (in the category of sheaves of sets) if and only if the induced maps on all the stalks are surjective.

2. Let $f : X \to Y$ be a map of topological spaces. Prove pushforward $f_*$ and pullback $f^{-1}$ for sheaves of sets form an adjoint pair of functors.

3. Let $j : U \to X$ be an open immersion. Show that $j^{-1}$ has a left adjoint $j_!$ on the category of sheaves of sets. Characterize the stalks of $j_!(G)$. (Hint: $j_!$ is called extension by zero when you do this for abelian sheaves...)

4. Let $F$ be an abelian sheaf on $X$. Show that $F$ is the quotient of a (possibly very large) direct sum of sheaves all of whose terms are of the form

$$j_!(\mathbb{Z}_U)$$

where $U \subset X$ is open and $\mathbb{Z}_U$ denotes the constant sheaf with value $\mathbb{Z}$ on $U$.

**Remark.** In the category of abelian sheaves the direct sum of a family of sheaves $\{F_i\}_{i \in I}$ is the sheaf associated to the presheaf $U \mapsto \oplus F_i(U)$. Consequently the stalk of the direct sum at a point $x$ is the direct sum of the stalks of the $F_i$ at $x$. 