Schemes

Excercises 1

Sheaves

1. Carefully prove that a map of sheaves of **sets** is an epimorphisms (in the category of sheaves of sets) if and only if the induced maps on all the stalks are surjective.

2. Let $f : X \to Y$ be a map of topological spaces. Prove pushforward f_* and pullback f^{-1} for sheaves of **sets** form an adjoint pair of functors.

3. Let $j : U \to X$ be an open immersion. Show that j^{-1} has a left adjoint $j_!$ on the category of sheaves of sets. Characterize the stalks of $j_!(\mathcal{G})$. (Hint: $j_!$ is called extension by zero when you do this for abelian sheaves...)

4. Let $\mathcal{F}$ be an abelian sheaf on X. Show that $\mathcal{F}$ is the quotient of a (possibly very large) direct sum of sheaves all of whose terms are of the form

$$j_!(\mathbb{Z}_U)$$

where $U \subset X$ is open and $\mathbb{Z}_U$ denotes the constant sheaf with value $\mathbb{Z}$ on U.

Remark. In the category of abelian sheaves the direct sum of a family of sheaves $\{\mathcal{F}_i\}_{i \in I}$ is the sheaf associated to the presheaf $U \mapsto \oplus \mathcal{F}_i(U)$. Consequently the stalk of the direct sum at a point x is the direct sum of the stalks of the $\mathcal{F}_i$ at x.