FREE AND VERY FREE MORPHISMS INTO A FERMAT HYPERSURFACE

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1. Introduction

Any smooth projective Fano variety in characteristic zero is rationally connected and hence contains a very free rational curve. In positive characteristic a smooth projective Fano variety is rationally chain connected. However, it is not known whether such varieties are separably rationally connected, or equivalently, whether they have a very free rational curve. This is an open question even for nonsingular Fano hypersurfaces. See [Kol96] as well as [Deb01].

In this paper we consider the degree 5 Fermat hypersurface

$$X \quad : \quad X_0^5 + X_1^5 + X_2^5 + X_3^5 + X_4^5 + X_5^5 = 0$$

in $\mathbb{P}^5$ over an algebraically closed field k of characteristic 2. Note that X is a nonsingular projective Fano variety.

Theorem 1.1. *Any free rational curve $\varphi : \mathbb{P}^1 \to X$ has degree ≥ 8 and there exists a free rational curve of degree 8. Any very free rational curve $\varphi : \mathbb{P}^1 \to X$ has degree ≥ 9 and there exists a very free rational curve of degree 9.*

This result, although perhaps expected, is interesting for several reasons. First, it is known that X is unirational, see [Deb01, Page 52] (the corresponding rational map $\mathbb{P}^4 \dashrightarrow X$ is inseparable). Second, in [Bea90], it is shown that every nonsingular hyperplane section of X is isomorphic to a Fermat hypersurface of dimension 3 and this property characterizes Fermat hypersurfaces among all hypersurfaces of degree 5 in characteristic 2. We believe that these facts single out the Fermat as a likely candidate for a counter example to the conjecture below; instead our theorem shows that they are evidence for it.

Conjecture 1.2. Nonsingular Fano hypersurfaces have very free rational curves.

A little bit about the method of proof. In Section 2 we translate the geometric question into an algebraic question which is computationally more accessible. In Sections 3, 4, and 5 we exclude low degree solutions by theoretical methods. Finally, in Sections 6 and 7 we explicitly describe some curves which are free and very free in degrees 8 and 9 respectively.

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2. The overall setup

In the rest of this paper k will be an algebraically closed field of characteristic 2 and X will be the Fermat hypersurface of degree 5 over k. Let $\varphi : \mathbb{P}^1 \to X$ be a nonconstant morphism. We will repeatedly use that every vector bundle on $\mathbb{P}^1$ is a direct sum of line bundles, see [Gro57]. Thus we can choose a splitting

$$\varphi^* T_X = \mathcal{O}_{\mathbb{P}^1}(a_1) \oplus \mathcal{O}_{\mathbb{P}^1}(a_2) \oplus \mathcal{O}_{\mathbb{P}^1}(a_3) \oplus \mathcal{O}_{\mathbb{P}^1}(a_4).$$

Recall that φ is said to be a *free curve* on X if $a_i \geq 0$ and φ is said to be *very free* if $a_i > 0$. Consider the following commutative diagram

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & & \\
 & \mathcal{O}_X \arrow[r, equal] \arrow[d] & \mathcal{O}_X \arrow[d] & & \\
0 \arrow[r] & E_X \arrow[r] \arrow[d] & \mathcal{O}_X(1)^{\oplus 6} \arrow[r] \arrow[d] & \mathcal{O}_X(5) \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & T_X \arrow[r] \arrow[d] & T_{\mathbb{P}^5}|_X \arrow[r] \arrow[d] & N_{X/\mathbb{P}^5} \arrow[r] & 0 \\
 & 0 & 0 & &
\end{tikzcd} \tag{2.0.1}$$

with exact rows and columns as indicated. We will call E_X the *extended tangent bundle of X*. The left vertical exact sequence determines a short exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^1} \to \varphi^* E_X \to \varphi^* T_X \to 0.$$

The splitting type of $\varphi^* E_X$ will consistently be denoted $(f_1, f_2, f_3, f_4, f_5)$ in this paper. Since $\mathrm{Hom}_{\mathbb{P}^1}(\mathcal{O}_{\mathbb{P}^1}(f), \mathcal{O}_{\mathbb{P}^1}(a)) = 0$ if $f > a$ we conclude that

(1) If $f_i \geq 0$ for all i, then φ is free.
(2) If $f_i > 0$ for all i, then φ is very free.

For the converse, note that the map $\mathcal{O}_{\mathbb{P}^1} \to \varphi^* E_X$ has image contained in the direct sum of the summands with $f_i \geq 0$. Hence, if $f_i < 0$ for some i, then φ is not free. Finally, suppose that $f_i \geq 0$ for all i. If there are at least two f_i equal to 0, then we see that φ is free but not very free. We conclude that

(3) If φ is free, then $f_i \geq 0$ for all i.
(4) If φ is very free, then either (a) $f_i > 0$ for all i, or (b) exactly one $f_i = 0$ and all others > 0.

We do not know if (4)(b) occurs.

Translation into algebra. Here we work over the graded k-algebra $R = k[S, T]$. As usual, we let $R(e)$ be the graded free R-module whose underlying module is R with grading given by $R(e)_n = R_{e+n}$. A *graded free R-module* will be any graded R-module isomorphic to a finite direct sum of $R(e)$'s. Such a module M has a *splitting type*, namely the sequence of integers $u_1, \ldots, u_r$ such that $M \cong R(u_1) \oplus \ldots \oplus R(u_r)$.

We will think of a degree d morphism $\varphi : \mathbb{P}^1 \to \mathbb{P}^5$ as a 6-tuple $(G_0, \ldots, G_5)$ of homogeneous elements in R of degree d with no common factors. Then φ is a morphism into X if and only if $G_0^5 + \ldots + G_5^5 = 0$. In this situation we define two graded R-modules. The first is called the *pullback of the cotangent bundle*

$$\Omega_X(\varphi) = \mathrm{Ker}(\tilde{\varphi} : R^{\oplus 6}(-d) \longrightarrow R)$$

where the map $\tilde{\varphi}$ is given by $(A_0, \ldots, A_5) \mapsto \sum A_i G_i$. The second is called the *the pullback of the extended tangent bundle*

$$E_X(\varphi) = \mathrm{Ker}(R^{\oplus 6}(d) \longrightarrow R(5d))$$

where the map is given by $(A_0, \ldots, A_5) \mapsto \sum A_i G_i^4$. Since the kernel of a map of graded free R-modules is a graded free R-module, both $\Omega_X(\varphi)$ and $E_X(\varphi)$ are themselves graded free R-modules of rank 5.

Lemma 2.1. *The splitting type of $\varphi^* E_X$ is equal to the splitting type of the R-module $E_X(\varphi)$.*

Proof. Recall that $\mathbb{P}^1 = \mathrm{Proj}(R)$. Thus, a finitely generated graded R-module corresponds to a coherent sheaf on $\mathbb{P}^1$, see [Har77, Proposition 5.11]. Under this correspondence, the module $R(e)$ corresponds to $\mathcal{O}_{\mathbb{P}^1}(e)$. The lemma follows if we show that $\varphi^* E_X$ is the coherent sheaf associated to $E_X(\varphi)$. Diagram (2.0.1) shows that $\varphi^* E_X$ is the kernel of a map $\mathcal{O}_{\mathbb{P}^1}(d)^{\oplus 6} \to \mathcal{O}_{\mathbb{P}^1}$ given by substituting $(G_0, \ldots, G_5)$ into the partial derivatives of the polynomial defining X. Since the equation is $X_0^5 + \ldots + X_5^5$, the derivatives are X_i^4, and substituting we obtain G_i^4 as desired. $\square$

3. Relating the Splitting Types

Observe that $\Omega_X(\varphi)$ is also a graded free module of rank 5 and so has a splitting type, which we denote using $e_1, \ldots, e_5$. In this section, we relate the splitting type of $\Omega_X(\varphi)$ to the splitting type of $E_X(\varphi)$.

If $(A_0, \ldots, A_5) \in \Omega_X(\varphi)$, then $A_0 G_0 + \cdots + A_5 G_5 = 0$ so that

$$A_0^4 G_0^4 + \cdots + A_5^4 G_5^4 = 0$$

by the Frobenius endomorphism in characteristic 2. Let

$$\mathcal{T} = \{(A_0^4, \ldots, A_5^4) \mid (A_0, \ldots, A_5) \in \Omega_X(\varphi)\}$$

in $E_X(\varphi)$. We denote the R-module generated by $\mathcal{T}$ as $R\langle\mathcal{T}\rangle$.

Lemma 3.1. *In the notation above, $E_X(\varphi) = R\langle\mathcal{T}\rangle$.*

Proof. Let $(B_0, \ldots, B_5)$ be an element of $E_X(\varphi)$ where B_i is a homogeneous polynomial of degree b. We consider the case $b \equiv 0 \mod 4$.

Observe that we can rewrite each monomial term of B_i as $(c^{1/4} S^\ell T^k)^4 S^i T^{4-i}$ or $(c^{1/4} S^\ell T^k)^4$ for some integers ℓ, k, where $c \in k$ and $0 < i < 4$. After collecting terms and applying the Frobenius endomorphism, we obtain

$$B_i = a_{i1}^4 + a_{i2}^4 S^3 T + a_{i3}^4 S^2 T^2 + a_{i4}^4 S T^3$$

where each a_{ij} is an element of R. Then, since $B_0 G_0^4 + \cdots + B_5 G_5^4 = 0$, substituting our expression for the B_i's and applying Frobenius we obtain

$$(\sum_{i=0}^{5} a_{i1} G_i)^4 + (\sum_{i=0}^{5} a_{i2} G_i)^4 S^3 T + (\sum_{i=0}^{5} a_{i3} G_i)^4 S^2 T^2 + (\sum_{i=0}^{5} a_{i4} G_i)^4 S T^3 \quad = \quad 0$$

The sums $\sum_{i=0}^{5} a_{ij} G_i$ are each themselves homogeneous polynomials. But since the degree of T in each term above is distinct modulo 4, the equation $\sum_{i=0}^{5} a_{ij} G_i = 0$ implies that $(a_{0j}, \ldots, a_{5j}) \in \Omega_X(\varphi)$, so that $(a_{0j}^4, \ldots, a_{5j}^4) \in \mathcal{T}$ for $1 \leq j \leq 4$.

Hence, every homogeneous element of $E_X(\varphi)$ is contained in the submodule generated by $\mathcal{T}$. Since the reverse containment is trivial, it follows that $E_X(\varphi) = R\langle \mathcal{T} \rangle$. The cases for $b \equiv 1, 2, 3 \mod 4$ follow similarly. $\square$

Proposition 3.2. *If $x_i = (x_{i0}, \ldots, x_{i5})$ for $1 \leq i \leq 5$ form a basis for $\Omega_X(\varphi)$, then $y_i = (x_{i0}^4, \ldots, x_{i5}^4)$ for $1 \leq i \leq 5$ form a basis for $E_X(\varphi)$.*

Proof. If $x_i \in \Omega_X(\varphi)$, then $y_i \in \mathcal{T}$ and every element of $\mathcal{T}$ is an R-linear combination of the y_i's. Since $E_X(\varphi) = R\langle \mathcal{T} \rangle$, every element of $E_X(\varphi)$ is also an R-linear combination of the y_i's so that the y_i's generate $E_X(\varphi)$. Moreover, $E_X(\varphi)$ is a free module of rank 5 over a domain, so the generators y_i for $E_X(\varphi)$ must also be linearly independent and hence form a basis. $\square$

Accounting for twist, a simple computation using the results above gives us the following.

Corollary 3.3. *If f_i denotes the splitting type of $E_X(\varphi)$ and e_i denotes the splitting type of $\Omega_X(\varphi)$, then for a degree d morphism, $f_i = 4e_i + 5d$.*

4. Numerology

We now utilize some facts about graded free modules in order to give constraints on potential splitting types. Given a graded free module

$$M = R(u_1) \oplus \ldots \oplus R(u_r)$$

one can observe that the Hilbert polynomial H_M is given by

$$H_M(m) = rm + u_1 + \ldots + u_r + r.$$

Let φ denote a free morphism into X. Noting that the map $\tilde{\varphi} : R(-d)_m^{\oplus n+1} \longrightarrow R_m$ is surjective for $m \gg 0$, we obtain

$$\begin{aligned} H_{\Omega(\varphi)}(m) &= \dim_k \left(\ker(R(-d)_m^{\oplus n+1} \longrightarrow R_m) \right) \\ &= (n+1)(-d+m+1) - (m+1) \\ &= nm + -d(n+1) + n. \end{aligned}$$

A similar calculation shows that,

$$H_{E_X(\varphi)}(m) = nm + d(n+1-5) + n$$

We continue to refer to the splitting type components of $\Omega(\varphi)$, respectively $E_X(\varphi)$ as e_i, respectively f_i. In both cases $n = r = 5$, so combining these two equations with the general form for the Hilbert polynomial of a graded free module, we obtain our first constraints:

$$e_1 + e_2 + e_3 + e_4 + e_5 = -6d \tag{4.0.1}$$

$$f_1 + f_2 + f_3 + f_4 + f_5 = d. \tag{4.0.2}$$

Recall from Section 2 that a curve is free, respectively very free if $f_i \geq 0$, respectively $f_i > 0$ for each i. Since $f_i = 4e_i + 5d$, it follows that

$$e_i \geq -\frac{5d}{4} \tag{4.0.3}$$

where strict inequality implies the curve is very free. With these two bounds, we can quickly observe a few facts about curves of different degrees.

Remark 4.1.

(1) There exist no free curves in degrees $1, 2, 3, 6$, and 7.
(2) Any free curve of degree not divisible by 4 must be very free.
(3) There are no very free curves in degrees 4 or 8.
(4) The $\Omega(\varphi)$ splitting type of a degree 4 free curve must be $(-5, -5, -5, -5, -4)$.
(5) The $\Omega(\varphi)$ splitting type of a degree 5 very free curve must be $(-6, -6, -6, -6, -6)$.

All of these observation follow directly from the two constraints. For example, in degree 6, $e_1 + e_2 + e_3 + e_4 + e_5 = -6d = -36$. However, each $e_i \geq \frac{-30}{4} = -7.5$. So even if each e_i is at best -7, the e_i cannot sum to -36.

The rest of the remarks follow in a similar manner. Note that one can glean even more information about these curves from the constraints, but the remarks listed above are sufficient for our purposes.

5. Degree 4 and 5 morphisms into X

We will now show that there are no free morphisms of degrees 4 or 5 into X. A morphism $\varphi = (G_0, ..., G_5)$, where each $G_i = \sum_{j=0}^{d} a_{ij} S^{d-j} T^j$ is a homogeneous polynomials of degree d, gives us a $6 \times (d + 1)$ matrix (a_{ij}). We will denote this matrix as M_φ.

Lemma 5.1. *If φ is a degree 4 or 5 free morphism into X, then M_φ has maximal rank.*

Proof. This follows from Remark 4.1 (4) and (5) by observing that for a degree d morphism into X, the transpose of M_φ is the matrix of the k-linear map $\tilde{\varphi}_d : (R(-d)^{\oplus 6})_d \to R_d$. □

Lemma 5.2.

(a) *There are no degree 4 free morphisms into X.*
(b) *There are no degree 5 free morphisms into X.*

Proof. (a) Assume a degree 4 free morphism $\varphi = (G_0, ..., G_5)$ exists. By the previous lemma, the 6×5 matrix $M_\varphi = (a_{ij})$ has maximal rank. Since permuting the $G_i's$ does not affect the splitting type of $E_X(\varphi)$, we can assume that the first 5 rows of M_φ are linearly independent over k. Then $\det((a_{ij})_{i \leq 4}) \neq 0$. Now consider the matrix $\overline{M_\varphi} = (a_{ij}^4)$. By the Frobenius endomorphism on k, $\det((a_{ij}^4)_{i \leq 4}) = \det((a_{ij})_{i \leq 4}))^4 \neq 0$, proving that $\overline{M_\varphi}$ has maximal rank as well.

Since $G_0^5 + ... + G_5^5 = 0$, computing the coefficients of $G_0^5 + ... + G_5^5$, we obtain for $0 \leq j \leq 4$

$$\sum_{i=0}^{5} a_{ij}^4 a_{i1} = 0 \quad \text{and} \quad \sum_{i=0}^{5} a_{ij}^4 a_{i3} = 0. \tag{5.2.1}$$

The kernel of the map $k^6 \to k^5$ given by right multiplication by the matrix $\overline{M_\varphi}$ has dimension 1 because $\operatorname{rank}(\overline{M}_\varphi) = 5$. By (5.2.1), $(a_{01}, a_{11}, ..., a_{51})$, $(a_{03}, a_{13}, ..., a_{53})$ $\in \ker(k^6 \to k^5)$, and since these 6-tuples are columns of M_φ, they are linearly independent over k. Then $\dim_k(\ker(k^6 \to k^5)) \geq 2$, a contradiction.

(b) Assume $\varphi = (G_0, ..., G_5)$ is a degree 5 free morphism. By the previous lemma, the matrix $M_\varphi = (a_{ij})$ has maximal rank, and is invertible. Thus $\overline{M_\varphi} = (a_{ij}^4)$ is invertible by the same argument above. Since $G_0^5 + ... + G_5^5 = 0$, computing the coefficients of the polynomial $G_0^5 + \cdots + G_5^5$, we get $\sum_{i=0}^5 a_{ij}^4 a_{i2} = 0$ for $0 \leq j \leq 5$. Thus, the product of the row matrix $(a_{02}, a_{12}, ..., a_{52})$ and the matrix $\overline{M_\varphi}$ is 0, which is impossible because $(a_{02}, a_{12}, ..., a_{52}) \neq 0$ and $\overline{M_\varphi}$ is invertible. $\square$

6. Computations for the degree 8 free curve

Let $\varphi : \mathbb{P}^1 \to \mathbb{P}^5$ be a morphism given by the 6-tuple

$$\begin{aligned}
G_0 &= S^7T \\
G_1 &= S^4T^4 + S^3T^5 \\
G_2 &= S^4T^4 + S^3T^5 + T^8 \\
G_3 &= S^7T + S^6T^2 + S^5T^3 + S^4T^4 + S^3T^5 \\
G_4 &= S^8 + S^7T + S^6T^2 + S^5T^3 + S^4T^4 + S^3T^5 + T^8 \\
G_5 &= S^8 + S^7T + S^6T^2 + S^5T^3 + S^4T^4 + S^3T^5 + S^2T^6 + ST^7.
\end{aligned}$$

One can check by computer or by hand that this curve lies on the Fermat hypersurface $X \subset \mathbb{P}^5$.

Due to twisting, the domain of the map $\tilde{\varphi} : R(-8)^{\oplus 6} \to R$ has its first nontrivial graded piece in dimension 8. The G_i are linearly independent over k, hence the kernel is trivial in dimension 8. The matrix for the map $\tilde{\varphi}_9 : R(-8)_9^{\oplus 6} \to R_9$ is

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}$$

where each direct summand of the domain has a basis $\{(S, 0), (0, T)\}$, of which we take six copies (for total dimension 12), and the range has basis given by the degree 9 monomials in S and T, ordered by increasing T-degree (for total dimension 10). This matrix has rank 10, which means that the map in degree 9 is surjective. By rank-nullity, two dimensions of the kernel live in degree 9; denote the generators by x_1, x_2. Surjectivity of $\tilde{\varphi}$ in degree 9 implies surjectivity in all higher degrees. A second application of rank-nullity gives $\dim_k \Omega(\varphi)_{10} = 7$. Four of the generators are inherited from the previous degree, taking the forms

$$x_1S, x_2S, x_1T, x_2T.$$

We conclude that there are three additional generators in degree 10. Therefore, the splitting type of $\Omega_X(\varphi)$ is $e_1, ..., e_5 = -10, -10, -10, -9, -9$, which translates to a splitting type for $E_X(\varphi)$ of $f_1, ..., f_5 = 0, 0, 0, 4, 4$, hence the curve is free.

7. A Very Free Rational Curve of Degree 9

We conclude by giving an example of a degree 9 very free curve lying on X. Let $\varphi : \mathbb{P}^1 \to \mathbb{P}^5$ be a morphism into the Fermat hypersurface given by the 6-tuple

$$\begin{aligned}
G_0 &= S^4T^5 \\
G_1 &= S^9 + S^8T + S^5T^4 \\
G_2 &= S^9 + S^4T^5 + ST^8 \\
G_3 &= S^9 + S^8T + S^4T^5 + S^3T^6 + S^2T^7 + ST^8 \\
G_4 &= S^9 + S^5T^4 + S^3T^6 + S^2T^7 + ST^8 + T^9 \\
G_5 &= S^7T^2 + S^6T^3 + S^5T^4 + S^3T^6 + S^2T^7 + ST^8 + T^9.
\end{aligned}$$

Let $e_1, ..., e_5$ again denote the splitting type of $\Omega_X(\varphi)$. As in Section 6, we know that $e_i \leq -9$. Since the G_i are linearly independent over k, $\dim_k(\Omega_X(\varphi)_9) = 0$. Next we claim that $\tilde{\varphi_{10}} : R_1^{\bigoplus 6} \to R_{10}$ is surjective. In fact, it can be checked that the $\tilde{\varphi}(b_i)$ span R_{10}, where the b_i are distinct basis elements of $R_1^{\bigoplus 6}$. It follows that $\tilde{\varphi_n} : R(-9)_n^{\bigoplus 6} \to R_n$ is surjective for $n \geq 10$. Hence,

$$\begin{aligned}
\dim_k(\Omega_X(\varphi)_{10}) &= \dim_k(R_1^{\bigoplus 6}) - \dim_k(R_{10}) = 1 \\
\dim_k(\Omega_X(\varphi)_{11}) &= \dim_k(R_2^{\bigoplus 6}) - \dim_k(R_{11}) = 6.
\end{aligned}$$

After reordering, this yields $(e_1, ..., e_5) = (-11, -11, -11, -11, -10)$, which corresponds to the splitting type $(1, 1, 1, 1, 5)$ of $E_X(\varphi)$, showing that φ is very free. This completes the proof of Theorem 1.1.

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