Exercise 20: Find a pair of conics intersecting in exactly 4 points.

Proof: We will work over $\mathbb{C}$. Consider the conic $C_F$ given by the homogeneous polynomial $F = YZ - X^2$. The affine part of this conic (i.e., when $Z = 1$) is just the parabola $Y = X^2$ (at least over $\mathbb{R}$). Consider the conic $C_G$ given by the polynomial $G = (X+Y-Z)(-X+Y-Z) = (Y-Z)^2 - X^2$. The affine part of this conic are the two lines $X+Y-1 = 0$ and $-X+Y-1 = 0$. Note that the polynomials $F$ and $G$ have no common irreducible factors ($F$ itself is irreducible over $\mathbb{C}$). So, in particular Bezout’s theorem applies, and the conics $C_F$ and $C_G$ have at most 4 intersection points counting with multiplicities.

In fact, the affine parts of these conics intersect at 4 distinct points over $\mathbb{R}$, the points of intersection being $\left(\frac{-1+\sqrt{5}}{2}, \left(\frac{-1+\sqrt{5}}{2}\right)^2\right)$, $\left(\frac{-1-\sqrt{5}}{2}, \left(\frac{-1-\sqrt{5}}{2}\right)^2\right)$, $\left(\frac{1-\sqrt{5}}{2}, \left(\frac{1-\sqrt{5}}{2}\right)^2\right)$, $\left(\frac{1+\sqrt{5}}{2}, \left(\frac{1+\sqrt{5}}{2}\right)^2\right)$. So, the conics intersect in $\mathbb{P}^2$, at the 4 distinct points $\left[\frac{-1+\sqrt{5}}{2} : \left(\frac{-1+\sqrt{5}}{2}\right)^2 : 1\right]$, $\left[\frac{-1-\sqrt{5}}{2} : \left(\frac{-1-\sqrt{5}}{2}\right)^2 : 1\right]$, $\left[\frac{1-\sqrt{5}}{2} : \left(\frac{1-\sqrt{5}}{2}\right)^2 : 1\right]$, $\left[\frac{1+\sqrt{5}}{2} : \left(\frac{1+\sqrt{5}}{2}\right)^2 : 1\right]$. 
