Our main result is a lower bound for the Bergman kernel (on the diagonal) of an arbitrary pseudoconvex domain in $C^n$ in terms of volume of sublevel sets of the pluricomplex Green function. It is proved using the Donnelly-Fefferman estimate for the dbar-equation (which, as noticed by Berndtsson, easily follows from the Hörmander estimate) and the tensor-power trick. This lower bound for the Bergman kernel seems very accurate: it gives in particular another proof of the one-dimensional Suita conjecture. For convex domains in $C^n$ using Lempert’s theory one can show that the Bergman kernel is bounded from below by the reciprocal of the volume of the Kobayashi indicatrix. The latter can be used to simplify the Nazarov proof of the Bourgain-Milman inequality from convex analysis.