In the past 10 years we observed an increasing interest in the PDE community on equations that involve fractional powers of the Laplace operator in the whole space. This operator is a classical object in Harmonic Analysis and Functional Analysis. Nevertheless, some fine tools needed in the study of the PDEs like Harnack inequalities and Schauder regularity estimates are not available from the general theory. L. Caffarelli and L. Silvestre introduced a powerful tool to handle the fractional Laplacian known as the extension problem. This turned out to be a mayor breakthrough in the theory of nonlocal equations because thanks to it many problems could be attacked by using known techniques from usual PDE theory.

We will explain a novel point of view to handle fractional equations: the heat semigroup language approach. The method was introduced in my PhD thesis. With this we can understand what a fractional operator is. We can also generalize the Caffarelli-Silvestre extension problem to fractional powers of many differential operators, like the Laplacian on the torus, or more generally, the Laplace-Beltrami operator on a manifold, or the discrete Laplacian on a lattice. From this point on it is possible to prove regularity estimates for fractional nonlocal equations. We will show how to address these questions in the particular case of fractional elliptic equations in divergence form, which is a joint work with Luis Caffarelli.