In the last 15 years, White and Huisken-Sinestrari developed a far-reaching structure theory for the mean curvature flow of mean convex hypersurfaces. Their papers provide a package of estimates and structural results that yield a precise description of singularities and of high curvature regions in a mean convex flow. In the present talk, we explain a new treatment of the theory of mean convex (and k-convex) flows. This includes: (1) an estimate for derivatives of curvatures, (2) a convexity estimate, (3) a cylindrical estimate, (4) a global convergence theorem, (5) a structure theorem for ancient solutions, and (6) a partial regularity theorem. Our new proofs are both more elementary and substantially shorter than the original arguments. Our estimates are local and universal. A key ingredient in our new approach is the new non-collapsing result of Andrews. Some parts are also inspired by the work of Perelman. This is joint work with Bruce Kleiner.