HONORS COMPLEX VARIABLES, MATH W4065
PROBLEM SET 1

DUE MONDAY, SEPTEMBER 21, 2009

(1) (Bak and Newman, problem 1.5) Suppose P is a polynomial with real coefficients. Show that $P(z) = 0$ if and only if $P(\overline{z}) = 0$ (i.e., zeroes of real polynomials come in complex conjugate pairs).
(2) (Bak and Newman, problem 2.3) Show that no non-constant analytic polynomial can take only imaginary values.
(3) (Bak and Newman, problem 2.5 and 2.6) Show that if f and g are both complex differentiable at z and $g(z) \neq 0$, so are $f + g$, fg, and f/g. Find the derivatives. Deduce that analytic polynomials are differentiable.
(4) (Needham, problem 4.2) The map $z \mapsto z^3$ acts on an infinitesimal shape. The image shape has been rotated by π, and is a factor of 12 larger. Where was the shape originally located? (There are two possibilities.)
(5) (Needham, problems 4.5) Consider the map $f(x + iy) = x^2 + y^2) + i(y/x)$. Find and sketch the curves that are mapped by f to horizontal lines and the curves that are mapped to vertical lines. Note that these meet at right angles. Nevertheless, show that f is not complex differentiable.

    Bonus: Show that no choice of real function $v(x, y)$ can make $f(x + iy) = (x^2 + y^2) + iv(x, y)$ complex differentiable.
(6) (From Stein and Shakarchi, problem 1.3) With $w = se^{i\theta}$, where s is a positive real number, find all solutions to the equation $z^n = w$ where n is a positive integer. How many solutions are there? Plot the solutions for $w = -1$, $n = 4$ and for $w = 1$, $n = 6$.