HONORS COMPLEX VARIABLES, MATH W4065
PROBLEM SET 9

DUE WEDNESDAY, NOVEMBER 25, 2009

(1) (Bak–Newman, 9.7) Find the Laurent expansion for
   (a) $\frac{1}{z^4+z^2}$ around $z = 0$,
   (b) $\frac{\exp(1/z^2)}{z-1}$ around $z = 0$,
   (c) $\frac{1}{z^2-4}$ around $z = 2$.

(2) Evaluate $\sum_{n=1}^{+\infty} \frac{1}{n^4}$.

(3) Evaluate $\sum_{n=1}^{+\infty} \frac{1}{n^2+1}$.

(4) (Stein–Shakarchi, 5.6.2) Find the order of growth of the following functions:
   (a) $f(z) = p(z)$ where p is a polynomial,
   (b) $f(z) = e^{bz^n}$, where $b \neq 0$,
   (c) $f(z) = e^{e^z}$.

(5) (Stein–Shakarchi, 5.6.11) Suppose that $f(z)$ is an entire function of finite order that misses two points. (That is, there are $a, b \in \mathbb{C}$ so that for all z, $f(z) \neq a$ and $f(z) \neq b$.) Show that f is constant. (*Hint:* If $f(z) \neq a$, then there is a function $g(z)$ so that $f(z) = a + e^{g(z)}$.)