Honors Complex Variables, Math W4065
Mid-term exam

Due October 26, 2009, at 4:10 PM

This exam is open notes and open book: you may use any of your notes, the notes posted online, and any of the three textbooks for this course. (Remember that all textbooks are available on reserve from the math library.) No other aids are permitted, and you may not discuss the content of the exam with anyone except me (for clarifications).

The exam is due at 4:10 PM on Monday. If you are late for class, the exam is late.

This exam is run under the terms of the Barnard Honor Code.

> We, the students of Barnard College, resolve to uphold the honor of the College by refraining from every form of dishonesty in our academic life. We consider it dishonest to ask for, give, or receive help in examinations or quizzes, to use any papers or books not authorized by the instructor in examinations, or to present oral work or written work which is not entirely our own, unless otherwise approved by the instructor. We consider it dishonest to remove without authorization, alter, or deface library and other academic materials. We pledge to do all that is in our power to create a spirit of honesty and honor for its own sake.

Please sign the statement below, and hand it in with your exam.

> I pledge that I have upheld the Barnard Honor Code during this exam. I have neither given nor received help during this test.

Signed: ______________________________

Name: ______________________________

1. Prove that $f(z)$ is holomorphic near z_0 if and only if $\overline{f(\overline{z})}$ is holomorphic near $\overline{z_0}$.

2. Find $\oint \bar{z}\, dz$ around

   (a) The boundary of an arbitrary rectangle and

   (b) The boundary of an arbitrary circle.

3. Let $f(z) = e^z$. Find the Fourier expansion for $f(e^{i\theta})$.

4. Suppose $f(z)$ is holomorphic for $|z| < 1$ and $|f(z)| \le 1/(1 - |z|)$. Find the best upper bound for $|f^{(n)}(0)|$ that you can. (You will get some credit if you give any bound.)

5. Suppose that $f(z)$ is a meromorphic function on all of $\mathbb{C}$, and that $\lim_{z\to\infty} f(z) = \infty$. (Specifically, this means that for every $r > 0$, there is an $R > 0$ so that for $|z| > R$, $|f(z)| > r$.)

   (a) Consider the function $g(z) = 1/f(1/z)$. Show that $g(z)$ has an isolated singularity at 0, and classify it.

   (b) Show that $f(z)$ has only a finite number of poles.

   (c) Show that $f(z)$ is a ratio of two polynomials: there are polynomials $P(z)$, $Q(z)$ so that

   $$f(z) = \frac{P(z)}{Q(z)}$$

   when $f(z)$ is defined. (Such a ratio of two polynomials is called a *rational function.*)