

MATH W4065: HONORS COMPLEX VARIABLES
BARNARD COLLEGE, FALL 2009

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Mathematics 614

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Office hours: Tuesday 4-5PM, Thursday 10-11AM, or by appointment.

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Review section: Wednesday 5:30-6:30PM, Milbank 328

INTRODUCTION TO THE COURSE

In this course, we will learn about one-variable complex analysis. This is the study of complex functions of one complex variable. It is simpler and more elegant than the study of functions of two real variables, and in some ways even simpler than the study of functions of one real variable. For instance, a complex function with one continuous first derivative automatically has an infinite number of derivatives, so a whole class of pathological examples in real analysis have no analogues in the complex case.

We will study functions with complex derivatives. One key fact is that, considered as maps from the complex plane to itself, these functions preserve angles (as long as the derivative is not zero). As a result, they are often called **conformal** maps or functions. (“Conformal” comes from Latin roots meaning “same shape”.)

The real strength of complex analysis comes from properties of integrals of complex-analytic functions along paths. In particular, these path integrals remain unchanged when the path is perturbed; this gives a great deal of flexibility when evaluating integrals. A fundamental theorem, the **Cauchy residue theorem**, reduces the integral of an analytic function along a closed path to counting certain singularities in the region bounded by the path. This is quite powerful, and, indeed, many real integrals can most easily be done using this theorem.

We will next turn to the **Riemann mapping theorem**: given any two regions in the plane (satisfying some mild assumptions), there is always a conformal bijection between them. This has wide-ranging applications, from random walks to drawing maps of the surface of the brain.

Finally, we wrap up the semester with the **Gamma and zeta functions**, the next two most important functions after the exponential and trigonometric functions (sin, cos, etc.). The Gamma function extends the factorial function, and the zeta function is extremely useful in number theory. In particular, the zeta function can be used to prove the **prime number theorem**, on how many primes there are less than some bound. This also illustrates some further techniques in complex analysis:

- **Function engineering**, finding functions which are zero or singular at specified places. Often these are unique.
- **Functional equations**, which determine the behavior of a function in one place from the location in another.

HOW DOES THIS COURSE DIFFER FROM V3007?

While there will be a lot of overlap between “Complex Variables” and “Honors Complex Variables”, there will also be several differences. In particular, this course will:

- Present somewhat more material than V3007. For instance, the Riemann mapping theorem is typically not covered in V3007.
- Work in a more abstract setting, with less of an eye to applications, than V3007.
- Be more rigorous and proof-based. We unavoidably need a few results from other courses (which will be stated without proof), but we will keep those to a minimum.

Consequently, the material in W4065 will be presented at a somewhat higher pace than in V3007.

You should strongly consider W4065 instead of V3007 if:

- You are planning to major in mathematics.
- You enjoy thinking about (abstract?) mathematical questions.
- You are planning to take more advanced mathematics courses later.
- You want a deeper background in mathematics for your major or other studies.

You should not take W4065 if:

- You already took V3007.
- You are only taking complex variables because of a (non-mathematics) degree requirement.

PREREQUISITES

You should be familiar with real analysis (e.g., Modern Analysis I), or alternatively have taken calculus and be familiar with how to write proofs .

TOPICS

We will follow Bak and Newman’s *Complex Analysis* (required), with additional topics taken from Needham’s *Visual Complex Analysis* and Shakarchi and Stein’s *Complex Analysis*. These second two books are not required, but you may find it helpful to browse them at the appropriate time. All books are on reserve in the Mathematics library.

Topics will include:

- Introduction to complex analysis.
- Complex differentiability: What does it mean for a function to have a complex derivative? An explicit differential equation (the Cauchy-Riemann equations).
- Visualizing complex maps: conformality (preserving angles) and examples.
- Integrals of complex differentiable functions and invariance under deformation of the path of integration.
- One derivative implies infinitely many. Power series expansions and convergence.
- Möbius transformations.
- Harmonic functions.
- Evaluating definite integrals by counting poles enclosed (the Cauchy residue theorem). Applications to real integrals.
- Conformal mappings and the Riemann mapping theorem.
- The Gamma and zeta functions.
- The prime number theorem.

HOMEWORKS, EXAMS, AND GRADING

There are weekly homework assignments, due each Monday (starting September 21). Homeworks (and other assignments) will be marked down by 5% for every day they are late, and will not be accepted more than a week late. Collaboration with other students is strongly encouraged, you are required to turn in solutions to the homework carefully written and put in final form by yourself. Also, any resource you use other than the textbooks must be cited in your homework. This includes electronic resources (including Wikipedia and Google) and human resources (including the help room and your classmates). Failure to cite sources constitutes academic misconduct.

There is also a 5–10 paper, due on December 3. This term paper would further explore a topic related to the course, to get you experience in reading and explaining mathematics on your own. A problem statement is due two weeks in advance, and a draft is due one week in advance. More information will be distributed later.

There will be a take-home midterm due on October 26.

The grade for the course will be calculated as follows:

- Homework: 20%
- Paper: 25%
- Midterm: 20%
- Class participation, bonus problems: 5%
- Final exam: 30%

HONOR CODE

This is a Barnard class. Unlike Columbia, Barnard has an honor code, which you are expected to follow in this course. It states:

We, the students of Barnard College, resolve to uphold the honor of the College by refraining from every form of dishonesty in our academic life. We consider it dishonest to ask for, give, or receive help in examinations or quizzes, to use any papers or books not authorized by the instructor in examinations, or to present oral work or written work which is not entirely our own, unless otherwise approved by the instructor. We consider it dishonest to remove without authorization, alter, or deface library and other academic materials. We pledge to do all that is in our power to create a spirit of honesty and honor for its own sake.

