Maximal grade for HW4: 50 points

Section 2.3: 28. (10 points) Compute the limit: \( \lim_{h \to 0} \frac{(3 + h)^{-1} - 3^{-1}}{h} \).

Solution 1: Let us simplify the expression in the limit:

\[
\lim_{h \to 0} \frac{(3 + h)^{-1} - 3^{-1}}{h} = \lim_{h \to 0} \frac{\frac{1}{3 + h} - \frac{1}{3}}{h} = \lim_{h \to 0} \frac{3 - (3 + h)}{h \cdot 3(3 + h)} = \\
\lim_{h \to 0} \frac{-1}{h} \cdot \frac{3}{3(3 + h)} = -\frac{1}{3}.
\]

Solution 2: Let \( f(x) = x^{-1} \), then \( f'(x) = (-1)x^{-2} \), and

\[
\lim_{h \to 0} \frac{(3 + h)^{-1} - 3^{-1}}{h} = f'(3) = -3^{-2} = -\frac{1}{9}.
\]

Section 2.7: 18. (10 points) Find an equation of the tangent line to the graph of \( y = g(x) \) at \( x = 5 \) if \( g(5) = -3 \) and \( g'(5) = 4 \).

Solution: Since the slope of the tangent line equals \( g'(5) = 4 \), its equation has a form \( y = 4x + b \). Since the line should pass through the point \((5, g(5)) = (5, -3)\), we can write an equation for \( b \): \(-3 = 4 \cdot 5 + b\), and \( b = -23 \). Therefore the tangent line is given by the equation \( y = 4x - 23 \).

Section 3.1: 34. (10 points) Find an equation of the tangent line to the curve \( y = x^4 + 2x^2 - x \) at the point \((1, 2)\).

Solution: We have \( y'(x) = 4x^3 + 4x - 1 \), so \( y'(1) = 7 \). Since the slope of the tangent line equals \( y'(1) = 7 \), its equation has a form \( y = 7x + b \). Since the line should pass through the point \((1, 2)\), we can write an equation for \( b \):
2 = 7 \cdot 1 + b, and b = -5. Therefore the tangent line is given by the equation \( y = 7x - 5 \).

52. (10 points) For what values of \( x \) does the graph of \( f(x) = e^x - 2x \) have a horizontal tangent?

Solution: We have \( f'(x) = e^x - 2 \), and the horizontal tangent corresponds to \( f'(x) = 0 \), so \( e^x = 2 \) and \( x = \ln 2 \).

70. (10 points) Where is the function \( h(x) = |x - 1| + |x + 2| \) differentiable? Give a formula for \( h' \) and sketch the graph of \( h \) and \( h' \).

Solution: For \( x \geq 1 \) we have \( h(x) = x - 1 + x + 2 = 2x + 1 \), for \(-2 \leq x < 1 \) we have \( h(x) = 1 - x + x + 2 = 3 \), for \( x < -2 \) we have \( h(x) = 1 - x - x - 2 = -2x - 1 \). In short, we can write

\[
h(x) = \begin{cases} 
2x + 1 & \text{if } x \geq 1, \\
3 & \text{if } -2 \leq x < 1, \\
-2x - 1 & \text{if } x < -2.
\end{cases}
\]

Therefore

\[
h'(x) = \begin{cases} 
2 & \text{if } x > 1, \\
0 & \text{if } 2 < x < 1, \\
-2 & \text{if } x < -2.
\end{cases}
\]

Note that \( h'(x) \) is undefined for \( x = 1 \) and for \( x = -2 \), since the left and right derivatives do not agree.
Figure 1: Graph of $h(x)$ for problem 70

Figure 2: Graph of $h'(x)$ for problem 70