MATH 2030
Midterm 1
February 19, 2015

Name: ________________________________

UNI: ________________________________

DO NOT OPEN THIS EXAM YET

(1) Fill in your name, UNI and section number.
(2) This exam is closed-book and closed-notes; no calculators, no phones.
(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.
(4) You may continue your solutions on additional sheets of paper provided by the proctor. If you do so, please write your name and UNI at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.
(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.
(6) Show your work; correct answers only will receive only partial credit (unless noted otherwise).
(7) Be careful to avoid making grievous errors that are subject to heavy penalties.
(8) If you need more blank paper, ask a proctor.

Out of fairness to others, please stop working and close the exam as soon as the time is called. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

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1. (20 points) For the differential equation $y' = y^2 - 3y + 2$:
   a) Find all equilibrium solutions
   b) Sketch the graphs of various types of solutions
   c) Find the limit $\lim_{t \to +\infty} y(t)$ depending on the initial condition $y(0) = y_0$
   d) Solve the equation explicitly
2. Solve the differential equations:
   a) \( xy' = 2y + 1 \)

   b) \( (1 + t^2)y' + 2ty = 1 \)
3. Find all values of the parameter $a$ such that the differential equation
\[(x + y + 1) + (ax + y + 2)y' = 0.\]

is exact. Solve the equation for these values of $a$. 

4. Solve the initial value problems:
   a) \( y' = \sqrt{y}, y(0) = 1 \)
   b) \( ty' + 3y = 2t^2 + 1, y(1) = 1 \)
5. Find all values of the parameter $r$ such that all solutions of the differential equation

$$y' = ry + r^2 t$$

are bounded for $t > 0$. 
This is a bonus problem. Please start this problem only if you completed the rest of the exam.

6*. Show that the solution of the initial value problem

\[ y' = x^2 + y^2 - 1, \quad y(0) = 0.5 \]

has an inflection point. *Hint: sketch the graph of this solution first*