Problem 3. Let \( \vec{q}(t) = (2-t, 1+t, 3-2t) \) and \( \vec{r}(t) = (1+\ln(t), 2, t^3) \) be the positions of two particles. At what time do the particles collide? Where do they collide? At what angle do they collide?

The particles collide if they are in the same position at the same time: \( \vec{q}(t) = \vec{r}(t) \).

\[
\begin{align*}
    (2-t, 1+t, 3-2t) &= (1+\ln t, 2, t^3) \\
    2-t &= 1+\ln t \\
    1+t &= 2 \\
    3-2t &= t^3
\end{align*}
\]

So \( 2-t = 1+\ln t \), \( 1+t = 2 \), and \( 3-2t = t^3 \).

There actually is a collision at \( t = 1 \):

\[
\begin{align*}
    2-1 &= 1+\ln 1 \quad \checkmark \\
    1+1 &= 2 \quad \checkmark \\
    3-2 &= 1^3 \quad \checkmark
\end{align*}
\]

The point where they collide is given by the position of either particle at \( t = 1 \) (after all, they are colliding): \( \vec{F}(1) = \vec{q}(1) = (1, 2, 1) \).

The angle at which they collide is the angle between their directions of motion, which are \( \vec{q}'(1) \) and \( \vec{r}'(1) \). \( \vec{q}'(1) = \langle 1, 1, -2 \rangle \) and \( \vec{r}'(1) = \langle 1/2, 0, 3t^2 \rangle \).

\[
\begin{align*}
    \vec{q}'(1) &= \langle -1, 1, -2 \rangle \\
    \vec{r}'(1) &= \langle 1/2, 0, 3 \rangle
\end{align*}
\]

So \( \theta = \cos^{-1}\left( \frac{\vec{q}'(1) \cdot \vec{r}'(1)}{|\vec{q}'(1)||\vec{r}'(1)|} \right) = \cos^{-1}\left( \frac{-1(1) + 1(0) + -2(3)}{\sqrt{1^2+1^2+(-2)^2} \sqrt{(1/2)^2+0^2+3^2}} \right) = \cos^{-1}\left( \frac{-7}{\sqrt{160}} \right) \).