Let $f$ be defined on $[a, b]$. Suppose $f$ is differentiable at $x \in (a, b)$ and $f$ has a maximum at $x$. Show that $f'(x) = 0$ by showing that

$$\lim_{h \to 0^+} \frac{f(x + h) - f(x)}{h} \leq 0$$

and

$$\lim_{h \to 0^-} \frac{f(x + h) - f(x)}{h} \geq 0.$$