The goal of this seminar is to learn the recent advances on the applications of derived category techniques and stability to the study of certain families of Fano manifolds and their associated hyperkähler manifolds.

The first part of the seminar (consisting approximately of four talks) will be a review of derived category theory: in particular, we will cover the basic definitions (exceptional objects, semiorthogonal decompositions, Kuznetsov components) and the main examples (projective spaces, Grassmannians, quadrics, cubic fourfolds, Gushel–Mukai manifolds, Debarre-Voisin manifolds, homogeneous spaces). Basic references for this part are in the work of Kuznetsov and Perry [Kuz15, Kuz14, Kuz16, KP18a, KP18b]. For the basics on derived category theory, we refer to [Huy06].

In the second part (roughly four talks), we will review stability and moduli spaces for Kuznetsov components. We will start by proving a fundamental result by Abramovich and Polishchuk [AP06] on how to base change t-structures, then we will cover the basic of Bridgeland stability [Bri07], including the latest definition from [BLM+19], and Bridgeland deformation theorem. The examples will be on K3 categories: we will review the general theory, by following the original paper [Bri08], and then the existence result for more general K3 categories [BLMS17, PPZ19]. Finally, the last lecture will be on the basics of moduli spaces of complexes, to arrive to the recent foundational paper [AH-LH18].

The last part of the seminar (two/three talks) will be on applications to cubic fourfolds and, if time permits, Gushel–Mukai manifolds, by first reviewing relative stability and moduli spaces from [BLM+19]

Tentative Plan.

**Friday, October 18, 2019:** Exceptional collections, semiorthogonal decompositions, examples (Beilinson’s theorem, Grassmannians and Lefschetz decompositions, quadrics, projective bundles, blow-ups). Refs: [Kuz14, Fon13, Kuz15]. Introductory talk to the basic definitions, through classical examples of exceptional collections; review also Kuznetsov’s definition of Lefschetz decomposition, and treat Fonarev’s example of Lefschetz decomposition on Grassmannians. Speaker: Nicolas Perrin.

**Friday, October 25, 2019:** Examples of Fano manifolds of Calabi-Yau type: cubics, Gushel-Mukai, Debarre-Voisin, and Iliev-Manivel manifolds. Refs: [BD85, DK18, DV10,
IM11]. Introductory talk to basic examples of Fano manifolds of Calabi-Yau type. Speaker: Frederic Han.

Friday, November 8, 2019: Kuznetsov components: Serre functors and examples. Construct and study semiorthogonal decompositions in the examples from the previous talk; prove Kuznetsov’s Serre functor theorem. Refs: [Kuz15]. Speaker: Céline Bonandrini.

Wednesday, November 13, 2019: Examples of derived equivalences of Kuznetsov components with K3s: $C_{14}$, $C_8$, and Gushel–Mukai. Refs: [AL17, Kuz10, KP18a, KP18b]. Speaker: Vladimiro Benedetti.

Friday, November 22, 2019: Base change for derived categories and t-structures. Refs: [Kuz11, AP06, BLM+19]. Speaker: Emanuele Macrì.

Friday, December 13, 2019: Stability conditions: definition and Bridgeland deformation theorem. Refs: [Bri07, Bay16, BMS16, BLM+19]. Speaker: Claire Voisin.

Wednesday, December 18, 2019: Stability conditions on K3 categories. Refs: [Bri08, BLMS17, PPZ19]. Speaker: Jieao Song.

Friday, January 10, 2020: Applications of Bridgeland stability on K3 surfaces to algebraic geometry: Mukai’s program. Refs: [ABS14, Fey17]. Speaker: Enrico Arbarello.


References


