Each question is worth 10 points. There are 5 questions, for a total of 50 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Prove that if \( y(x) = u(x) + iv(x) \) is a solution to the equation
\[
y'' + p(x)y' + q(x)y = 0
\]
then so are \( u(x) \) and \( v(x) \).

(2) Find a fundamental set \( y_1, y_2 \) of real-valued solutions for the equation
\[
y'' - 2y' + 2y = 0.
\]
Check, using the Wronskian, that \( y_1, y_2 \) indeed form a fundamental set of solutions.

(3) Use the method of undetermined coefficients to solve the IVP
\[
y'' + 3y' + 2y = 7 \sin x + \cos x, \quad y(0) = 0, \quad y'(0) = 1.
\]
Explain your ansatz.

(4) Let \( \alpha, \beta \) be real constants. The following equation is known as the Cauchy–Euler equation:
\[
x^2y'' + \alpha xy' + \beta y = 0.
\]
Show that the change of variables
\[
t = \ln x
\]
transforms it into a constant-coefficient equation. Use this to find the general (real-valued) solution to the Cauchy–Euler equation when \( \alpha = \beta = 1 \).

(5) Find the general solution to
\[
y''' - 2y'' - y' + 2y = 0.
\]