This final exam contains 2 parts. Part A contains 10 multiple choice problems and each counts for 5 points. Part B contains 5 problems. You have three hours to do them. Try first those problems you can do. You can ask the proctor for extra scratch papers.
PART A. This part contains 10 multiple choice problems. Each problem has only one correct answer. Circle the correct answer. No points will be given if you circle two or more answers. Each problem counts for 5 points. There is no penalty for choosing wrong answers. You can do your work on blank space, however, it will NOT count toward partial credits.

A1. The arc length of the curve \( r(t) = (\cos t, \cos t, \sqrt{2} \sin t) \) for \( t \) between 0 and \( 2\pi \) is
   (a) \( 4\pi \).
   (b) \( 2\sqrt{2}\pi \).
   (c) \( 2\pi \).
   (d) \( \pi \).

A2. The area of the region inside the cardioid \( r = 1 - \cos \theta \) for \( 0 \leq \theta \leq 2\pi \) is
   (a) \( \pi \).
   (b) \( 2\pi \).
   (c) \( 3\pi \).
   (d) \( \frac{3\pi}{2} \).

A3. Which one below is a normal vector to the plane that contains the point \((1, 0, 2)\) and the line \( \mathbf{r}(t) = \langle t, 1 + t, 2 - t \rangle \)?
   (a) \( \langle 1, 1, 2 \rangle \).
   (b) \( \langle 1, 1, -2 \rangle \).
   (c) \( \langle -1, 1, 2 \rangle \).
   (d) \( \langle -1, 1, -2 \rangle \).

A4. Let \( L \) be the intersection of the planes: \( x + 3y + z = 1 \) and \( x + 3y - z = 1 \). Which one below is a vector parallel to the line \( L \)?
   (a) \( \langle 3, -1, 0 \rangle \).
   (b) \( \langle 1, 1, 1 \rangle \).
   (c) \( \langle 2, 1, 2 \rangle \).
   (d) \( \langle 1, 3, 1 \rangle \).
A5. The function \(z\) of \(x\) and \(y\) is defined by the equation \(xe^y + ye^z + ze^x = 0\). The partial derivative \(\partial z/\partial x\) at the point \((0, 0, 0)\) is
(a) \(1 + e\).
(b) \(1/e + e\).
(c) 1.
(d) \(-1\).

A6. The directional derivative of the function \(f(x, y) = e^x \sin y\) along the direction \(\frac{1}{\sqrt{5}} \langle -1, 2 \rangle\) at the point \((1, \pi/4)\) is equal to
(a) \(e/\sqrt{10}\).
(b) \(e/\sqrt{5}\).
(c) \(-e/\sqrt{10}\).
(d) \(-e/\sqrt{5}\).

A7. The maximum rate of change (among all directions) of \(f(x, y) = 2xy - x^3\) at the point \((1, 1)\) is
(a) 4.
(b) \(\sqrt{3}\).
(c) \(\sqrt{13}\).
(d) \(\sqrt{5}\).

A8. Let \(E\) be the tangent plane of the surface \(z - \cos x \cos y = 0\) at the point \((\pi/4, \pi/4, 1/2)\). Which one is a normal vector to the plane \(E\)?
(a) \(\langle 1, 1, -2 \rangle\).
(b) \(\langle 1, -1, 1 \rangle\).
(c) \(\langle 1, 1, 2 \rangle\).
(d) \(\langle 0, 1, 0 \rangle\).

A9. Find the length of the cardioid \(r = 1 + \sin \theta\).
(a) 4.
(b) 8.
(c) \(4\pi\).
(d) \(8\pi\).

A10. Let \(C\) be the intersection of the ellipsoid \(4x^2 + 2y^2 + z^2 = 16\) and the plane \(y = 2\). The tangent line of the curve \(C\) at the point \((1, 2, 2)\) is
(a) \(x = 1, y = 2 + t, z = 2 - 2t\).
(b) \(x = 1 + t, y = 2, z = 2 + 2t\).
(c) \(x = 1 + 2t, y = 2, z = 2 - t\).
(d) \(x = 1 + t, y = 2, z = 2 - 2t\).
PART B. This part contains 5 problems. Show all work to obtain credits. Unsupported answers will not earn any partial credits.

B1. Let \( u = x^2y + y^2z^2 \), where \( x = rs\cos t \), \( y = rs^2e^{-t} \), and \( z = r^2s\sin t \). Find the value of \( \frac{\partial u}{\partial s} \) when \( r = 2 \), \( s = 1 \), and \( t = 0 \).

B2. (a) Find a parametric equation of the intersection \( C \) of \( x = y \) and \( x^2 + \frac{z^2}{2} = 1 \).
   (b) Find the curvature \( \kappa \) of the curve \( C \) at the point \( (\sqrt{2}/2, \sqrt{2}/2, 1) \).
B3. For the function $f(x, y) = 3x^3 - x + y^2 - y$, (a) Find all its critical points; (b) Find its absolute maximum and minimum values in the domain $D = \{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq 1\}$. 
B4. Find the extreme values of the function $f(x, y, z) = 5x + 3y + z$ with constraints $x + y + z = -3$, $x^2 + y^2 + z^2 = 5$. 
B5. Find the tangent line of the intersection of spheres: $x^2 + y^2 + z^2 = 5, (x - 2)^2 + (y - 2)^2 + (z - 2)^2 = 5$ at the point $(2, 1, 0)$. 