Instructions.

1. Show all the steps of your work clearly.

2. No textbooks, notes or calculators allowed.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Your Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>100 + 10</td>
<td></td>
</tr>
</tbody>
</table>
1. At what points is $f$ not continuous?

2. At what points is $f$ not differentiable?

3. What is $f''(\frac{1}{2})$?

4. What is $\int_{-2}^{1} f(x)dx$?

5. Is the function $g(x) = \int_{-2}^{x} f(t)dt$ continuous on $[-2, 1]$? Why?
2) (10 points)

1. (5 points) Use the definition of the derivative to evaluate \( f'(x) \) for \( f(x) = \frac{1}{x^2} \).

2. (5 points) Use your answer from (1) to estimate \( \frac{1}{3.99^2} \).
3) (8 points) The volume of a cube is increasing at a rate of 10cm³/min. How fast is the surface area increasing when the length of an edge is 3cm?
4) (8 points) Find two positive numbers such that sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.
5) (6 points) Evaluate the following limit by any means.

\[ \lim_{x \to 0^+} \sin(x)^x \]
6) (10 points) Given the definite integral $\int_{0}^{3} (\sqrt{9 - x^2} - (x - 3)) \, dx$.

1. (5 points) Write it as a limit of right Riemann sums. DO NOT EVALUATE.

2. (5 points) Evaluate $\int_{0}^{3} (\sqrt{9 - x^2} - (x - 3)) \, dx$
   
   Hint: Interpret this as an area.
7) (10 points)

1. (5 points) Let \( g(x) = \int_{3}^{\tan(x)} \frac{e^t}{t+1} \, dt \). What is the derivative of \( g(x) \), i.e. \( g'(x) \)?

2. (5 points) Evaluate the following indefinite integral

\[ \int \frac{\sin x \cos x + \cos x}{\sin^2 x + 1} \, dx \]
8)(10 points) Determine if the following statements are true or false. If it is true, please explain. If it is false, please provide an counterexample.

1. Differentiable functions are integrable.

2. If $f$ is continuous on the interval $[a, b]$, then $\frac{d}{dx} \int_a^b f(x) dx = f(x)$.

3. For all $n$, we have $\frac{d}{dx} |x^n| = n |x|^{n-1}$.

4. $f(x) = x^2 - 10x + 1$ has no roots in the interval $[0, 2]$.

5. Suppose that $f$ is continuous on $[0, 4]$, $f(0) = 1$ and $2 \leq f'(x) \leq 5$ for all $x$ in the interval $(0, 4)$, then $f(4)$ satisfies $9 \leq f(4) \leq 21$. 
9)(14 points)
Let \( R \) be the finite region bounded by the graphs of \( y = 3e^x \) and \( y = 2x + 8 \) on the interval \([0, 1]\). Use definite integrals to find the following quantities.

1. The area of \( R \).

2. The volume of the solid obtained when \( R \) is revolved around the horizontal line \( y = -5 \).
3. The volume of the solid obtained when \( R \) is revolved around the vertical line \( x = 9 \).

NOTE: Only set up the integral in this question. DO NOT EVALUATE.
10)(14 points) Find the below information for the function

\[ f(x) = (x - 2)e^{-x} \]

and use it to sketch its graph on the last page.

1. Domain

2. Horizontal and vertical asymptotes (if any)

3. Zeros of \( f(x) \)
4. Intervals on which \( f(x) \) is increasing/decreasing

5. Local maxima and minima

6. Intervals on which \( f(x) \) is concave up/concave down

7. Points of inflection
8. Graph of $f(x)$
Bonus (10 points): For what values of $c$ is there a straight line that intersects the curve

$$y = x^4 + cx^3 + 12x^2 - 5x + 2$$

in four distinct points?