

Algebraic topology, Fall 2013

Homework 3, due Wednesday, September 25

From Hatcher:

Section 1.2 exercises 15, 16 (page 54).

Section 1.3 exercises 5, 9, 10 (pages 79-80).

Section 1.4 exercises 2, 12 (pages 358-359).

1. Describe explicitly a universal covering space of the subspace of \mathbb{R}^3 that is the union of a sphere and a diameter.
2. Prove that if CW complex X has no cells of dimension $1, 2, \dots, n$, then $\pi_i(X) = 0$ for $i \leq n$.
3. (a) Assuming that $\pi_2(\mathbb{S}^2) = \mathbb{Z}$ compute $\pi_2(\mathbb{S}^2 \vee \mathbb{S}^2)$ (hint: embed this space into a certain larger space). (b) Can you determine π_2 of the wedge $\mathbb{S}^2 \vee \mathbb{S}^1$?

Discussion. Sample problems: Section 1.3 exercises 6, 20.

1. Show that the join $X * Y$ of two nonempty spaces X and Y is simply-connected if X is path-connected.