Algebraic topology, Fall 2013

Homework 4, due Wednesday, October 2

From Hatcher:
Section 4.1 exercises 11, 14 (pages 358-359).

1. (a) Prove that the long sequence of a pair (X, A)

$$\ldots \longrightarrow \pi_n(A) \xrightarrow{i_*} \pi_n(X) \xrightarrow{j_*} \pi_n(X, A) \xrightarrow{\partial} \pi_{n-1}(A) \longrightarrow \ldots$$

is exact.

(b) Assuming A is a retract of X, show that i_* is a monomorphism, j_* is an epimorphism, $\partial = 0$ and $pi_n(X) = \pi_n(X, A) \oplus \pi_n(A)$.

(c) Do a similar analysis for the case when A is contractible to a point in X.

2. Prove the 5-lemma (in the case when maps f_1, f_2, f_4, f_5 are isomorphisms).

3. Using the fiber bundle $\mathbb{S}^\infty \longrightarrow \mathbb{CP}^\infty$ with fiber $\mathbb{S}^1$ compute homotopy groups of $\mathbb{CP}^\infty$ and explain why it is an Eilenberg-Maclane space.