Algebraic topology, Fall 2013

Homework 5, due Wednesday, October 9

1. Show that $\mathbb{S}^2$ and $\mathbb{S}^3 \times \mathbb{CP}^\infty$ have isomorphic homotopy groups but are not homotopy equivalent.

2. Hatcher Section 4.1 exercise 15 on page 359.

3. Show that the Whitehead product $[\alpha, \beta] = \alpha\beta\alpha^{-1}\beta^{-1}$ if $\alpha, \beta \in \pi_1(X)$.

4. Prove that for $\alpha \in \pi_n(X), \beta \in \pi_m(X)$

$$[\alpha, \beta] = (-1)^{nm}[\beta, \alpha]$$

for $\alpha \in \pi_n(X), \beta \in \pi_m(X)$.

5. Exercise 10.6 on page 81 from Botvinnik's "Lecture notes on Algebraic Topology" (there's a link to it in "Additional resources" section of our webpage).

6. (Exercise 10.10 from Botvinnik's notes) Prove that the suspension $\Sigma(S^n \times S^k)$ is homotopy equivalent to the wedge

$$S^{n+1} \vee S^{k+1} \vee S^{n+k+1}.$$