Lie groups

Homework #3, due Monday, March 19.

1. Determine the multiplicities of irreducible representations L_λ in induced representations I_μ of the symmetric group S_4, for all partitions $\lambda, \mu \vdash 4$ by counting semistandard tableaux. Also determine multiplicities of L_λ in induced representations I_μ^-, for all $\lambda, \mu \vdash 4$.

2. Interpret the number of standard λ-tableaux, for a partition $\lambda \vdash k$, in the language of $GL(n)$-representations.

3. Which irreducible representations of S_n have the property that they remain irreducible upon restriction to S_{n-1}? What about restriction to S_{n-2}?

4. Find multiplicities of irreducible representations of $GL(4)$ in the tensor products $V(2\epsilon_1 + 2\epsilon_2) \otimes V(3\epsilon_1)$, $V(2\epsilon_1 + 2\epsilon_2) \otimes V(\epsilon_1 + \epsilon_2 + \epsilon_3)$. Are these tensor products multiplicity-free?

5. Determine the multiplicities of weight spaces of the irreducible representation of $GL(6)$ with the highest weight $3\epsilon_1 + 2\epsilon_2$ (for each Weyl group orbit on weight spaces you can compute just one multiplicity).

6. Modify the proof of the Jacobi-Trudi formula obtained in class for the complete symmetric functions to the elementary symmetric functions case. The proof can also be found in Bruce Sagan's book The Symmetric Group, Section 4.5, available via SpringerLink.