18 a) \[
\begin{pmatrix}
1 & 2 & 0 & 2 & 0 \\
0 & 0 & 1 & 3 & 0 \\
0 & 0 & 1 & 4 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\] not in RREF form
- pivots are circled
- there is a non-zero
- above pivot in column 3

b) \[
\begin{pmatrix}
0 & 0 & 2 & 0 & 3 \\
0 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\] yes, this is an RREF matrix

c) \[
\begin{pmatrix}
1 & 2 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 \\
\end{pmatrix}
\] not in RREF
- 2nd row is 0, should be at
- the bottom of the matrix

d) \[
\begin{pmatrix}
0 & 1 & 2 & 3 & 4 \\
\end{pmatrix}
\] yes, this is in RREF
\[ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot (-1) + 2 \cdot 2 + 3 \cdot 1 \\ 2 \cdot (-1) + 3 \cdot 2 + 4 \cdot 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \]

**#22** If a solution is unique, there are no free variables, each variable is leading, and there is a pivot in each column of the RREF. Since there are 3 equations and 3 unknowns, there is no room for zero rows.
RREF of coefficient matrix is the identity matrix
\[
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]
RREF of augmented matrix can have any entries in the augmented row
\[
\begin{pmatrix} 1 & 0 & \vdots & a_1 \\ 0 & 1 & \vdots & a_n \end{pmatrix}
\]
\(a_1, \ldots, a_n\) any numbers.

**#28** The rank is the number of pivots (and number of leading variables). Since we have 3 columns and the rank is 3, there must be a pivot in each column, and the matrix is the identity matrix, with some number of zero rows at the bottom.
\[ A \vec{e}_1 = \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ d \\ g \end{pmatrix}, \text{ the first column of } A \]

\[ A \vec{e}_2 = \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} b \\ e \\ h \end{pmatrix}, \text{ the second column of } A \]

\[ A \vec{e}_3 = \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c \\ f \\ k \end{pmatrix}, \text{ the third column of } A. \]