Modern Algebra I, Spring 2017

Homework 13 (Last one!), due Wednesday April 26 before class.

Gallagher §12 covers solvable groups, Judson briefly discusses them around Example 13.19. Gallagher in §19 proves that a group of order less than 60 cannot be non-abelian simple. For counting problems, follow the blueprint from Wednesday's lecture. The orbit counting is also explained in Judson 14.3.

1. (a) Show that a non-abelian simple group cannot be solvable.

(b) We proved in class that A_5 is simple. Use this and other results (from Gallagher §12) to show that groups S_5, S_6 are not solvable.

2. Show that the direct product $G \times H$ of two solvable groups is solvable. (Hint: how can you construct the required chain of subgroups of $G \times H$ from those for G and H?)

3. (20 points) Prove that there are no simple groups of order (a) 70, (b) 64, (c) 100, (d) 65, (e) 96, (f) 80. (Hint: use methods from Gallagher §19.)

4. Determine the number of ways to color vertices of a regular pentagon using 4 colors, up to the symmetries of the pentagon (the symmetry group is D_5). First derive the formula when the number of colors is n, and then specialize to $n = 4$.

5. (a) How many necklaces can you arrange out of ten red and two green beads, up to dihedral symmetries?

(b) Same question, but out of six red, three green, and three blue beads.

6.